Unit #10: Applications of Differentiation
Topic: Applications of Differentiation Review
Objective: SWBAT repair skills needed to complete exam on derivative applications including Mean Value Theorem, first/second derivative tests, and graph analysis.

Part I: Read each question carefully and choose the correct answer. BE CAREFUL!!!

1) If $f(x) = \frac{1}{2}x + sinx$ is defined on the closed interval $[0, \pi]$, then f has a critical value at $x =$

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{5\pi}{6}$ (e) none of these

2) For what value(s) of x is the function $f(x) = 1 + x^2 - 2x^4$ increasing?

(a) $-1 < x < 1$

(b) $x < -\frac{1}{2}\ and\ 0 < x < \frac{1}{2}$

(c) $-\frac{1}{2} < x < 0\ and\ x > \frac{1}{2}$

(d) $x < -1\ and\ x > 1$

(e) $x > 0$

3) Determine the value of c that satisfies Rolle's Theorem for the function $f(x) = 2x^2 - 8$ on $[-2,2]$.

(a) -1 (b) 0 (c) 1 (d) $\frac{1}{2}$ (e) $-\frac{1}{2}$

4) If $f''(x) = x^2(x+2)(x-3)$, then the graph of function f has a point of inflection for what value(s) of x?

(a) -2 and 3 only (b) 3 and 0 only (c) $0, -2,$ and 3

(d) -2 only (e) 3 only

5) For what value of k will $f(x) = x + \frac{k}{x}$ have a relative maximum at $x = -2$?

(a) -4 (b) -2 (c) 2 (d) 4 (e) none of these

6) An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(a) $y = -6x - 6$ (b) $y = -3x + 1$ (c) $y = 3x - 1$

(d) $y = 2x + 10$ (e) $y = -4x + 1$

7) A polynomial function $p(x)$ has a relative maximum at $(-2,4)$, a relative minimum at $(1,1)$, a relative maximum at $(5, 7)$ and no other critical points. How many zeroes does $p(x)$ have?

(a) one (b) two (c) three (d) four (e) none

8) The function $g(x) = x^4 - 4x^2$ has

(a) one relative minimum and two relative maximums
(b) one relative minimum and one relative maximum
(c) no relative minimum and two relative maximums
(d) two relative minimums and no relative maximums
(e) two relative minimums and one relative maximum

9) The graph of $y = x^4 + 8x^3 - 72x^2 + 4$ is concave down for

(a) $-6 < x < 2$ (b) $x > 2$ (c) $x < -6$

(d) $x < -3 - 3\sqrt{5}$ or $x > -3 + 3\sqrt{5}$ (e) none of these

10) For what value(s) of x is the graph of $f(x) = 6x^2 - x^3$ increasing and concave up?

(a) $0 < x < 2$ (b) $x > 4$ (c) $2 < x < 4$ (d) $x < 0$ (e) $x > 2$

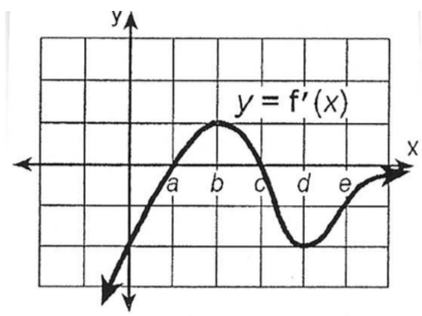
11) Which of the following is the absolute maximum value of $f(x) = \frac{5 + 2x}{x^2 + 14}$ on the interval $[-2,4]$?

(a) $\frac{1}{18}$ (b) $\frac{13}{30}$ (c) $\frac{8}{7}$ (d) $\frac{1}{2}$ (e) none of these

12) For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum?

(a) 10 (b) 4 (c) 3 (d) -4 (e) -10

13) The graph shown below shows the derivative f' of the function f.



At what value(s) of x does the function f have a point of inflection?

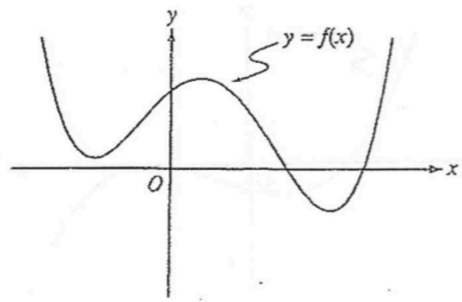
(a) c and e only (b) a, b, c, and d only (c) a and c only

(d) b and d only (e) a only

14) Given function f defined by $f(x) = (1 - x)^3$. What are all values of c, in the closed interval $[0, 3]$, that satisfy the conditions of the Mean Value Theorem?

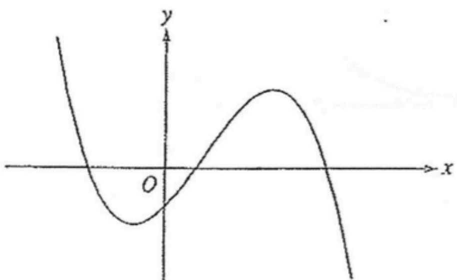
(a) 0 only (b) 2 only (c) 1 only (d) 0 and 2 (e) none of these

15)

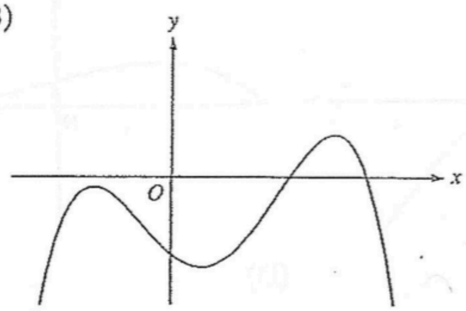


The graph of $f(x)$ is shown in the figure above. Which of the following could be the graph of $f'(x)$?

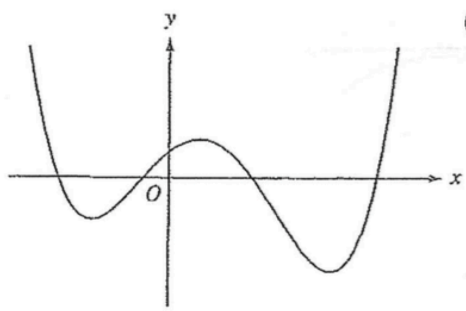
(A)



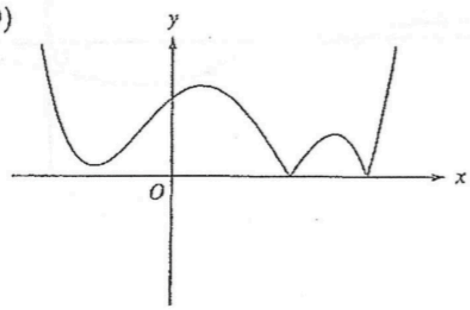
(B)



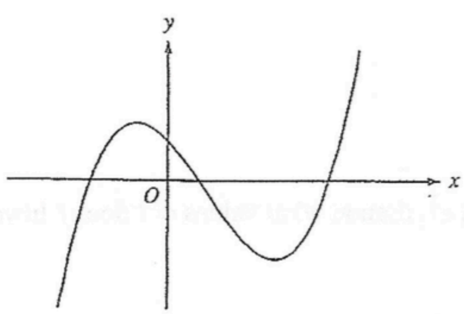
(C)



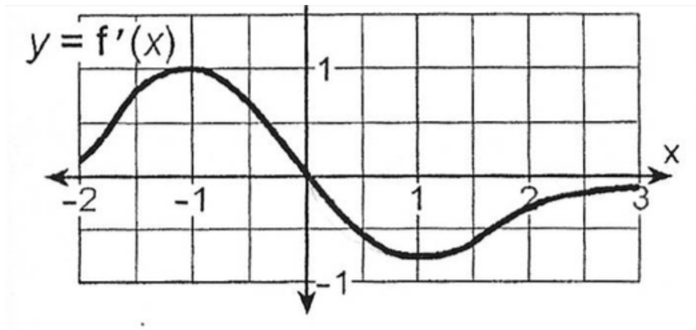
(D)



(E)



Questions 16-17 The graph shown below shows the derivative f' of the function f.



16) Determine the x −coordinate where $f(x)$ has a relative minimum.

(a) $x = 0$ (b) $x = 1$ (c) $x = -1$ (d) $x = 2$ (e) none of these

17) On which of the following intervals is f increasing and concave down?

(a) $(-2, -1)$ (b) $(-1, 0)$ (c) $(-1,1)$ (d) $(-2, 0)$ (e) $(0, 3)$

18) The graph of $f(x) = x^3 - 12x - 6$ is

(a) increasing for all x such that $x > -2$
(b) increasing for all real values of x
(c) increasing for all x such that $x < 2$
(d) decreasing for all real values of x
(e) decreasing for all x such that $-2 < x < 2$

Part II: Read each question carefully and show all work.

19) Show that 5 is a critical value of the function $g(x) = 2 + (x - 5)^3$, but g does not have a local extreme value at 5.

20) Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

(a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

(b) Find all intervals on which the graph of f is concave down. Justify your answer.

21) Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

22) Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

23) For $y = x^4 - 72x^2 - 17$, use analytic methods to find the exact intervals on which the function is: (Justify all answers)

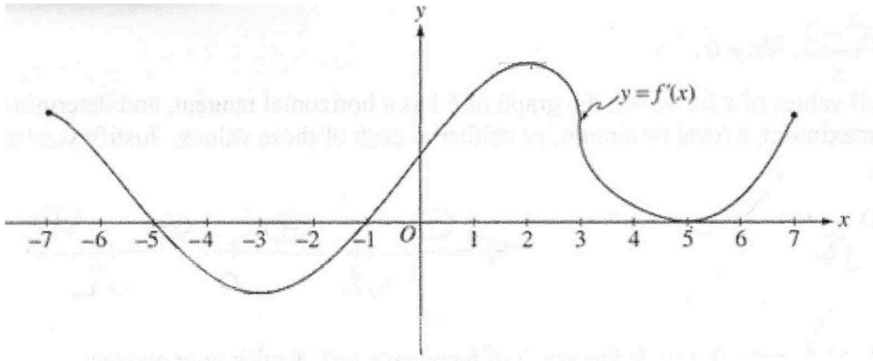
(a) increasing/decreasing

(b) local extreme values

(c) concave up/down

(d) points of inflection

24)



The figure above shows the graph of $f\,'$, the derivative of the function f, for $[-7,7]$.

(a) Find all values of x at which f attains a relative minimum or maximum. Justify your answers.

(b) Find all the values of x for which $f'' < 0$.

(c) Find all the values of x for which f has a point of inflection. Justify your answer.

Answer Key

1) A	2) B	3) B	4) A	5) D	6) B
7) B	8) E	9) A	10) A	11) D	12) A
13) D	14) B	15) E	16) E	17) B	18) E

19) No sign change	20) a) $x = 4$ is rel max b) $(0,6)$
21) a) $x = \pm\sqrt{2}$ is rel min b) all $\mathbb{R}$ except $x \neq 0$	22) a) $\left(y - \frac{2}{e^2}\right) = -\frac{1}{e^2}\left(x - e^2\right)$ b) rel max at $x = e$ c) $x = e^{\frac{3}{2}}$
23) a) incr $(-6,0)(6,\infty)$ decr $(-\infty,-6)(0,6)$ b) rel min $x = \pm 6$ rel max $x = 0$ c) concave up $(-\infty,\sqrt{12})$ $(\sqrt{12},\infty)$ concave down $(-\sqrt{12},\sqrt{12})$ d) $\pm\sqrt{12}$	
24) a) rel min $x = -1$ rel max $x = -5$ b) (-7,-3) (2,5) c) $x = -3, x = 2, x = 5$	