Unit #6: Continuity   Date: _____
Topic: Continuity Review
Objective: SWBAT repair the skills needed to answer questions about continuity and the definitions of the derivative.

NO CALCUATOR ALLOWED

Directions: Read each question carefully and show all work!

1) Let f be the function defined by the following:
$$\begin{cases} \sin x, & x < 0 \\ x^2, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ x - 3, & x \ge 2 \end{cases}$$

   For what values of x is the function NOT continuous?

2) For each of the following, determine any points of discontinuity and identify the type of discontinuity.

   a) $g(x) = \dfrac{2x^2 - 8x}{x^2 - 3x - 4}$

   b) $g(x) = \begin{cases} |2x - 3|, if\ x < 1 \\ x^2 - 2, if\ x \ge 1 \end{cases}$

3) For $(x) = \begin{cases} ax + 1, & x \le -2 \\ 5 + ax^2, & x > -2 \end{cases}$, find the value of a which makes the function continuous at $x = -2$.

4) If $(x) = \begin{cases} ax^2 - 3b, & x > 2 \\ -3, & x = 2 \\ 3bx - a, & x < 2 \end{cases}$, what values of a and b make the function continuous at $x = 2$?

5) a) If $f(x) = 2x^3 + x^2 - 4$, is there a number $x = c$ on the interval $[0,2]$ such that $f(c) = 3$. Justify your answer.

   b) If $f(x) = -3x^3 - 1$, is there a number $x = c$ on the interval $[1,3]$ such that $f(c) = 0$. Justify your answer.

6) For $f(x) = \begin{cases} x^2 + 1, & x < -3 \\ 2 - x, & x \geq -3 \end{cases}$, is the function continuous at $x = -3$? Justify your answer using the definition of continuity.

7) Give a formula for the extended function of $f(x) = \frac{3x^2 - 12}{7x - 14}$ that is continuous at $x = 2$. Show all work that leads to your answer.

8) A toy car travels on a straight path. During the time interval $0 \le t \le 60$ seconds, the toy car's velocity v, measured in feet per second, is a continuous function. Selected values are given below.

$t(sec)$	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-10	-15	-10	-7	-5	0	13

Is there a time t from $0 \le t \le 60$, such that $v(t) = -2$? Justify your answer.

9) Let $h(x)$ be a continuous function. Selected values of g are given in the table below.

x	2	3	5	8	14
$h(x)$	-1	0	6	5	-2

What is the least number of solutions the equation $h(x) = 3$ has on the closed interval $[2,14]$? Explain your reasoning.

10) Find the average rate of change of $f(x) = 4x^2 - 3x$ over the interval [-3,5].

11) Using the graph below, determine if each of the following statements is TRUE or FALSE.
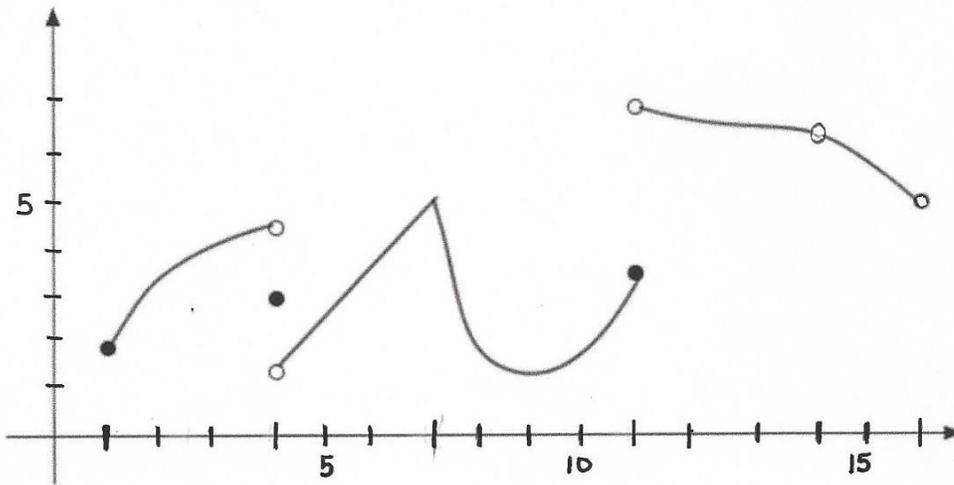
a) $f(x)$ is continuous on [1,4)   _____

b) $f(x)$ is differentiable at $x = 7$ and $x = 9$ _____

c) $f(x)$ is differentiable at $x = 11$ _____

d) $f(x)$ is continuous on (4,11]   _____

e) $f(x)$ is differentiable at $x = 2$ _____

f) $f(x)$ is continuous on (11,16)   _____

12) Given $y = \sqrt{x + 3}$

 a) Find the equation of the tangent line to the curve when $x = -2$.

 b) Find the equation of the normal line to the curve when $x = -2$.

13) If $f(x) = 3x^2 - 5$, find $f'(x)$ using

 a) the formal definition of the derivative.

 b) the alternate definition of the derivative

14) At $x = 3$, the function given by $f(x) = \begin{cases} x^2, x < 3 \\ 6x - 9, x \geq 3 \end{cases}$ is

    (a)   continuous, but not differentiable
    (b)   differentiable, but not continuous
    (c)   neither continuous nor differentiable
    (d)   both continuous and differentiable

15) Which of the following point(s) of discontinuity of $f(x) = \dfrac{3x + 6}{x^2 - 3x - 10}$ is/are not removable?

    (a) $x = -2$ (b) $x = -2 \ and \ 5$ (c) $x = 5$ (d) none of these

16) Find an equation of the line tangent to the curve $y = x^2 - x$, at the point (2,2).