Unit: Differential Equations
Topic: Differential Equations Review
Objective: SWBAT solve various problems by using differential equations.
Directions: Read each question carefully and show all work.

1. Fill in the appropriate letter slope field in the blanks below next to its matching differential equation.
a)

b)

c)


$$
-\frac{d y}{d x}=-\frac{y}{x}
$$

$$
\cdots \quad \frac{d y}{d x}=2-y
$$

$\cdots \quad \frac{d y}{d x}=\frac{y}{2}$
$-\frac{d y}{d x}=x+y$
2. If a substance decomposes at a rate proportional to the amount of the substance present, and if the amount decreases from 40 g to 10 g in 2 hr , then what is the constant of proportionality?
3. Polonium 210 decays into lead with a half-life of 138 days. How long will it take for $90 \%$ of the radioactivity in a sample of Polonium 210 to dissipate?
4. Suppose that the number of bacteria in a certain culture increases at a rate proportional to the number present, and that if $t$ is measured in minutes $\frac{d B}{d t}=\frac{1}{20} B$

How long will it take for the number of bacteria to double in size?
5. A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8,000 bacteria. When will the population reach 30,000 ?
6. Find $f(2)$ by solving the separable differential equation $\frac{d y}{d x}=2 x y^{2}$ with the intial condition $f(1)=1$.
7. Consider the differential equation $\frac{d y}{d x}=\frac{x+1}{y}$. Let $y=f(x)$ be a particular solution to this differential equation with the initial condition $f(0)=-2$.
(a) Use Euler's method with two steps of equal size, starting at $x=0$, to approximate $f(1)$. Show work that leads to your answer.
(b) Find the particular solution $y=f(x)$ to the given differential equation with the initial condition $f(0)=-2$.
8. A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 2 hours there are 4800 bacteria. At the end of 4 hours, there are 19,200 bacteria. How many bacteria were present intially?
9. Biologists stocked a lake with 400 fish and estimated the carrying capacity of the lake to be 10,000 . If the number of fish tripled in the first year, how long will it take for the population to increase to 5,000 ?
10. At midnight, with the temperature inside your house at $70^{\circ} \mathrm{F}$ and the temperature outside at $20^{\circ} \mathrm{F}$, your furnace breaks down. Two hours later, the temperature in your house has fallen to $50^{\circ} \mathrm{F}$. Assume that the outside temperature remains constant at $20^{\circ} \mathrm{F}$. At what time will the inside temperature of your house reach $40^{\circ} \mathrm{F}$ ?
11. Suppose a population of wolves grows according to the logistic differential equation $\frac{d P}{d t}=3 P-0.01 P^{2}$, where P is the number of wolves at time t , in years. Which of the following statements are true?
I. $\lim _{t \rightarrow \infty} P(t)=300$
II. The growth rate of the wolf population is greatest when $P=150$.
III. If $P>300$, the population of wolves is increasing.
(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III
12. (Calculator Permitted) A population of animals is modeled by a function $P$ that satisfies the logistic differential equation $\frac{d P}{d t}=0.01 P(100-P)$, where t is measured in years.
(a) If $P(0)=20$, solve for $P$ as a function of $t$.
(b) Use your answer to (a) to find $P$ when $t=3$ years. Give exact and 3-decimal approximation.
(c) Use your answer to (a) to find $t$ when $P=80$ animals. Give exact and 3-decimal approximation.
13. Solve each of the following differential equations so that $y>0$ :
a) $\frac{d y}{d x}=\frac{e^{2 x}}{4 y^{3}}$
b) $\frac{d y}{d x}=\frac{3 \ln x}{x y}$ and contains the point $(\mathrm{e}, 2)$

## Answer Key

1. B C
2. $\frac{\ln 1 / 4}{2}$ or -.693
3. 458.426
A D
4. $20 \ln 2 \approx 13.863 \mathrm{~min}$.
5. 4.430
6. $-1 / 2$
7. A) $-31 / 12$
B) $-\sqrt{x^{2}+2 x+4}$
8. 1200
9. 2.680
10. 3.587 hrs later or about $3: 30 \mathrm{am}$
11. C
12. A) $P=\frac{100}{1+4 e^{-t}}$
B) 83.392
C) $\ln 16 \approx 2.772$
13. A) $y=\sqrt[4]{\frac{1}{2} e^{2 x}+C}$
B) $y=\sqrt{3 \ln ^{2} x+1}$
