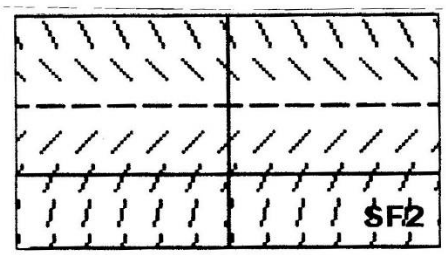
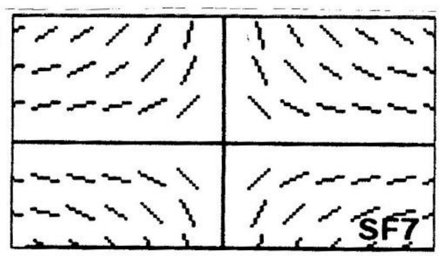
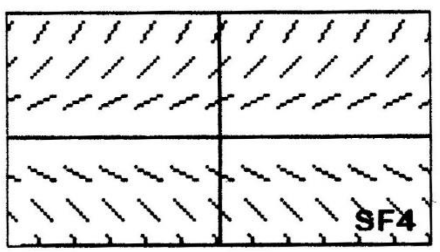
Unit: Differential Equations
Topic: Differential Equations Review
Objective: SWBAT solve various problems by using differential equations.

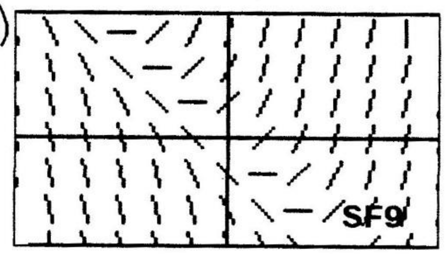
Directions: Read each question carefully and show all work.

1. Fill in the appropriate letter slope field in the blanks below next to its matching differential equation.

a)


b)


c)


d)


________ $\frac{dy}{dx} = -\frac{y}{x}$ ________ $\frac{dy}{dx} = \frac{y}{2}$

________ $\frac{dy}{dx} = 2 - y$ ________ $\frac{dy}{dx} = x + y$

2. If a substance decomposes at a rate proportional to the amount of the substance present, and if the amount decreases from 40g to 10g in 2 hr, then what is the constant of proportionality?

3. Polonium 210 decays into lead with a half-life of 138 days. How long will it take for 90% of the radioactivity in a sample of Polonium 210 to dissipate?

4. Suppose that the number of bacteria in a certain culture increases at a rate proportional to the number present, and that if t is measured in minutes

$$\frac{dB}{dt} = \frac{1}{20}B$$

How long will it take for the number of bacteria to double in size?

5. A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8,000 bacteria. When will the population reach 30,000?

6. Find $f(2)$ by solving the separable differential equation $\frac{dy}{dx} = 2xy^2$ with the intial condition $f(1) = 1$.

7. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(0) = -2$.

   (a) Use Euler's method with two steps of equal size, starting at $x = 0$, to approximate $f(1)$. Show work that leads to your answer.

   (b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

8. A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 2 hours there are 4800 bacteria. At the end of 4 hours, there are 19,200 bacteria. How many bacteria were present intially?

9. Biologists stocked a lake with 400 fish and estimated the carrying capacity of the lake to be 10,000. If the number of fish tripled in the first year, how long will it take for the population to increase to 5,000?

10. At midnight, with the temperature inside your house at 70°F and the temperature outside at 20°F, your furnace breaks down. Two hours later, the temperature in your house has fallen to 50°F. Assume that the outside temperature remains constant at 20°F. At what time will the inside temperature of your house reach 40°F ?

11. Suppose a population of wolves grows according to the logistic differential equation $\frac{dP}{dt} = 3P - 0.01P^2$, where P is the number of wolves at time t, in years. Which of the following statements are true?

I. $\lim_{t\to\infty} P(t) = 300$

II. The growth rate of the wolf population is greatest when $P = 150$.

III. If $P > 300$, the population of wolves is increasing.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

12. (Calculator Permitted) A population of animals is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = 0.01P(100 - P)$, where t is measured in years.

(a) If $P(0) = 20$, solve for P as a function of t.

(b) Use your answer to (a) to find P when $t = 3$ years. Give exact and 3-decimal approximation.

(c) Use your answer to (a) to find t when $P = 80$ animals. Give exact and 3-decimal approximation.

13. Solve each of the following differential equations so that $y > 0$:

a) $\frac{dy}{dx} = \frac{e^{2x}}{4y^3}$

b) $\frac{dy}{dx} = \frac{3lnx}{xy}$ and contains the point (e, 2)

Answer Key

1. B C
   A D

2. $\frac{ln^1/_4}{2}$ or -.693

3. 458.426

4. $20ln2 \approx 13.863min.$

5. 4.430

6. -1/2

7. A) $-31/12$ B) $-\sqrt{x^2+2x+4}$

8. 1200

9. 2.680

10. 3.587 hrs later or about 3:30 am

11. C

12. A) $P = \frac{100}{1+4e^{-t}}$ B) 83.392 C) $ln16 \approx 2.772$

13. A) $y = \sqrt[4]{\frac{1}{2}e^{2x} + C}$ B) $y = \sqrt{3ln^2x + 1}$