Key

Unit 3: Trigonometry
Topic: Trigonometry Review
Objective: SWBAT solve various problems using trigonometry.

NO CALCULATOR
Directions: Read each question carefully and show all work.

1) Determine the value of each of the following:

a) $\cos\left(-\frac{5\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$
Q2

b) $\csc\left(\frac{2\pi}{3}\right)$ $\frac{2\sqrt{3}}{3}$
Q2

c) $\sin\left(\frac{7\pi}{6}\right)$ $-\frac{1}{2}$
Q3

d) $\tan\left(-\frac{5\pi}{3}\right)$ $\sqrt{3}$
Q1

2) What are the coordinates on the unit circle for each of the following angles?

a) $\frac{4\pi}{3}$ $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
Q3

b) $\frac{7\pi}{4}$ $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
Q4

c) $-\frac{5\pi}{6}$ $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
Q3

d) $\frac{13\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{13\pi}{4}$
Q2

3) If an angle intersects the unit circle at the point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, what is the value for the sine and cosine of the angle? What could be the measure of the angle?

C S

Q2

$\frac{5\pi}{6}$

or $-\frac{7\pi}{6}, \frac{17\pi}{6}, \ldots$

4) Simplify each of the following expressions for the given angle.

a) $2sinxsecx(1-6tan^2x)$; $x=\frac{2\pi}{3}$ Q2

$\cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)\left(-\frac{2}{1}\right)\left(1-\frac{6}{1}\left(-\frac{\sqrt{3}}{1}\right)^2\right) = -2\sqrt{3}\,(1-18) = -2\sqrt{3}\,(-17)$

$= 34\sqrt{3}$

b)) $\frac{tant-tantsin^2t}{2sintcost}$; $t=-\frac{3\pi}{4}$

Q3

$\frac{1-(1)\left(-\frac{\sqrt{2}}{2}\right)^2}{\cancel{2}\left(-\frac{\sqrt{2}}{\cancel{2}}\right)\left(-\frac{\sqrt{2}}{2}\right)} = \frac{1-\frac{1}{2}}{\frac{2}{2}} = \frac{1}{2}$

c) $\frac{sec^2t-2cos^2t}{3cost-sint}$; $t=\frac{13\pi}{6}$

Q1

$\frac{\left(\frac{2}{\sqrt{3}}\right)^2-\frac{2}{1}\left(\frac{\sqrt{3}}{2}\right)^2}{3\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{2}\right)} = \frac{\frac{4}{3}-\cancel{2}\cdot\frac{3}{\cancel{4}\,2}}{\frac{3\sqrt{3}-1}{2}}$

$\frac{8-9}{6}$

$-\frac{1}{\cancel{6}\,3}\cdot\frac{2}{3\sqrt{3}-1} = \frac{-1}{9\sqrt{3}-3}$

5) Simplify each of the following:

a) $\frac{secx}{sinx}-\frac{sinx}{cosx}$

$\left(\frac{1}{cosx}\cdot\frac{1}{sinx}\right)-\frac{sinx}{cosx}$

$\frac{\overset{cos^2x}{1-sin^2x}}{\cancel{cosx}\,sinx} = \frac{cosx}{sinx} = cotx$

b) $\frac{sinx-sin^3x}{cos^3xsin^3x}$

$\frac{\cancel{\sin x}(1-\sin^2 x)}{\cos^{\cancel{3}\,5} x \sin^{\cancel{3}\,2} x} = \frac{1}{\cos x \sin^2 x}$

$\sec x \csc^2 x$

6) Prove each of the following: $1+\tan^2 x = \sec^2 x$

a) $\frac{1-sec^2x}{sec^2x-1} = cot^2x - csc^2x$

$\frac{-\tan^2 x}{\tan^2 x} = -1$

$= \cot^2 x - \csc^2 x$ ✓

* $1+\cot^2 x = \csc^2 x$

$\cot^2 x - \csc^2 x = -1$

OR

$\left(\frac{1-\sec^2 x}{1}\right)\left(\frac{1}{\tan^2 x}\right)$

$\frac{1-\sec^2 x}{1} \cdot \frac{\cos^2 x}{\sin^2 x}$

$\frac{\cos^2 x - 1}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x}$

$\cot^2 x - \csc^2 x$ ✓

b) $\frac{tan\theta csc^2\theta}{\underbrace{1+tan^2\theta}_{\sec^2 x}} = cot\theta$

$\frac{\cancel{\sin\theta}}{\cos\theta}\left(\frac{1}{\sin^{\cancel{2}}\theta}\right) \cdot \frac{\cos^{\cancel{2}} x}{1}$

$\frac{\cos x}{\sin x}$

$\cot x$ ✓

7) Solve each of the following equations for x, where $0 \leq x \leq 2\pi$:

a) $5\sqrt{3} - 10sinx = 0$

$-10\sin x = -5\sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}}$

b) $3sec^2x - 1 = 5$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

c) $tanxcotx + tanx = 0$

$\tan x(\cot x + 1) = 0$

$\tan x = 0$ $\quad$ $\cot x = -1$

$\boxed{x = 0, \pi, 2\pi}$ $\quad$ $\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$

d) $csc^2x - cscx = 2$

$(\csc x - 2)(\csc x + 1)$

$\csc x = 2$ $\quad$ $\csc x = -1$

$\sin x = 1/2$ $\quad$ $\sin x = -1$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$ $\quad$ $\boxed{x = \frac{3\pi}{2}}$

e) $2cos^2x + cosx - 1 = 0$

$(2\cos x - 1)(\cos x + 1)$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -1$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$ $\quad$ $\boxed{x = \pi}$

f) $5sin^2x - 3sinx + 1 = cos^2x$

$5\sin^2 x - 3\sin x + 1 = 1 - \sin^2 x$

$6\sin^2 x - 3\sin x = 0$

$3\sin x(2\sin x - 1) = 0$

$\sin x = 0$ $\quad$ $\sin x = 1/2$

$\boxed{x = 0, \pi, 2\pi}$ $\quad$ $\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$