

Unit #5: Improper Integrals

Topic: L'Hopital's Rule

Objective: SWBAT find the limit of a function using L'Hopital's Rule.

L'Hopital's Rule

L'Hopital's Rule allows us to use derivatives to evaluate limits that otherwise lead to indeterminate forms.

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$



Guillaume De l'Hôpital

1661-1704

Some examples of the indeterminate form $0/0$ are given below.

Warm Up #1:

1) Find each of the following limits.

a) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} =$

b) $\lim_{x \rightarrow 0} \frac{\sin x}{x} =$

c) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} =$

2) Find each of the following one-sided limits.

a) $\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} =$

b) $\lim_{x \rightarrow 0^-} \frac{x^2}{2x^2 + x} =$

When we reach a point where one of the derivatives approaches 0, as shown above, and the other does not, then

If the numerator approaches 0, we know the limit is _____.

If the denominator approaches 0, we know the limit is _____.
Now let's look at the indeterminate forms ∞/∞ , $\infty \cdot 0$, and $\infty - \infty$

Example #1: Find each of the following limits.

a) $\lim_{x \rightarrow \pi/2} \frac{\sec x}{1 + \tan x} =$

When we look at the form $\infty \cdot 0$ or $\infty - \infty$ we can make it look like $0/0$ or ∞/∞ and use L'Hopital's Rule.

b) $\lim_{x \rightarrow \infty} x \sin \frac{1}{x} =$

c) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right) =$

Problem Set #1: Find the limit for each of the following using L'Hopital's Rule.

1) $\lim_{x \rightarrow 0} \frac{\sin x - x}{2x^3} =$

2) $\lim_{x \rightarrow +\infty} \frac{x^2}{e^x} =$

3) $\lim_{x \rightarrow \pi/4} (1 - \tan x) \sec 2x =$

4) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right) =$

5) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} =$

6) $\lim_{x \rightarrow \infty} \frac{\ln x}{2\sqrt{x}} =$

7) $\lim_{x \rightarrow 0^-} \frac{\tan x}{x^2} =$

8) $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x}{7x^2 + 1} =$

9) $\lim_{x \rightarrow 0^+} (x \ln x) =$

10) $\lim_{x \rightarrow \pi} \frac{\csc x}{1 + \cot x} =$

Warm Up #2:

Find each of the following limits:

a) $\lim_{x \rightarrow 3} \frac{x-3}{x^2-2x-3} =$	b) $\lim_{x \rightarrow \infty} \frac{4-x^2}{4x^2-x-2} =$	c) $\lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{2}+h\right) - \cos\left(\frac{\pi}{2}\right)}{h}$
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Finally, let's look at the indeterminate forms 1^∞ , 0^0 , and ∞^0

Limits that lead to the indeterminate forms 1^∞ , 0^0 , and ∞^0 can sometimes be handled by taking logarithms first, then

$$\lim_{x \rightarrow a} \ln f(x) = L \quad \Rightarrow \quad \lim_{x \rightarrow a} e^{\ln f(x)} = e^L$$

Example #2: Find each of the following limits.

a) $\lim_{x \rightarrow 0} (1+x)^{1/x} =$

b) $\lim_{x \rightarrow 1} (x^2 - 2x + 1)^{x-1} =$

c) $\lim_{x \rightarrow \infty} x^{1/x} =$

Problem Set #2: Find each of the following limits.

11) $\lim_{x \rightarrow 0} (e^x + x)^{1/x} =$

12) $\lim_{x \rightarrow 0^+} (2x)^{x/4} =$

13) $\lim_{x \rightarrow 1^+} (x - 1)^{\ln x} =$

14) $\lim_{x \rightarrow \infty} (\ln x)^{1/x} =$

15) $\lim_{x \rightarrow 1} x^{1/(1-x)} =$

16) $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x =$

17) $\lim_{x \rightarrow \frac{\pi}{2}^-} (\sin x)^{\tan x} =$

18) $\lim_{x \rightarrow \infty} (e^x + 1)^{-2/x} =$

19) $\lim_{x \rightarrow 0} (1 + 2x)^{\csc x} =$

20) $\lim_{x \rightarrow \infty} (1 + 2x)^{1/2 \ln x} =$

Answer Key:

Problem Set #1

1. -1/12 2. 0 3. 1 4. 0 5. 2 6. 0 7. $-\infty$ 8. 5/7 9. 0 10. -1

Problem Set #2

11. e^2 12. 1 13. 1 14. 1 15. $1/e$ 16. e^3 17. 1 18. e^{-2} 19. e^2 20. \sqrt{e}