Unit #5: Improper Integrals
Topic: L'Hopital's Rule
Objective: SWBAT find the limit of a function using LHopital's Rule.

L'Hopital's Rule

L'Hopital's Rule allows us to use derivatives to evaluate limits that otherwise lead to indeterminate forms.

Guillaume De l'Hôpital

1661-1704

$$\boxed{\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{f'(a)}{g'(a)}}$$

Some examples of the indeterminate form $^0/_0$ are given below.

Warm Up #1:

1) Find each of the following limits.

a) $\lim_{x\to 1}\frac{x^2-1}{x-1}=$	b) $\lim_{x\to 0}\frac{sinx}{x}=$	c) $\lim_{x\to 0}\frac{1-cosx}{x}=$

2) Find each of the following one-sided limits.

a) $\lim_{x\to 0^+}\frac{sinx}{x^2}=$	b) $\lim_{x\to 0^-}\frac{x^2}{2x^2+x}=$

When we reach a point where one of the derivatives approaches 0, as shown above, and the other does not, then

If the numerator approaches 0, we know the limit is _______________________.

If the denominator approaches 0, we know the limit is ______________________.
Now let's look at the indeterminate forms $\boldsymbol{{}^{\infty}/_{\infty}}$, $\boldsymbol{\infty \cdot 0}$, ***and*** $\boldsymbol{\infty - \infty}$

Example #1: Find each of the following limits.

a) $\lim_{x \to \pi/2} \frac{secx}{1+tanx} =$

When we look at the form $\infty \cdot 0$ or $\infty - \infty$ we can make it look like $0/0$ or ${}^{\infty}/_{\infty}$ and use L'Hopital's Rule.

b) $\lim_{x \to \infty} xsin\frac{1}{x} =$

c) $\lim_{x \to 1}\left(\frac{1}{lnx} - \frac{1}{x-1}\right) =$

Problem Set #1: Find the limit for each of the following using L'Hopital's Rule.

1) $\lim_{x\to 0} \frac{sinx - x}{2x^3} =$	2) $\lim_{x\to +\infty} \frac{x^2}{e^x} =$
3) $\lim_{x\to \pi/4} (1 - tanx)sec2x =$	4) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{sinx}\right) =$
5) $\lim_{x\to 0} \frac{e^{2x} - 1}{x} =$	6) $\lim_{x\to \infty} \frac{lnx}{2\sqrt{x}} =$
7) $\lim_{x\to 0^-} \frac{tanx}{x^2} =$	8) $\lim_{x\to \infty} \frac{5x^2 - 3x}{7x^2 + 1} =$
9) $\lim_{x\to 0^+} (xlnx) =$	10) $\lim_{x\to \pi} \frac{cscx}{1 + cotx} =$

Warm Up #2:

Find each of the following limits:

a) $\lim_{x\to 3} \frac{x-3}{x^2-2x-3} =$	b) $\lim_{x\to\infty} \frac{4-x^2}{4x^2-x-2} =$	c) $\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}\right)}{h}$

Finally, let's look at the indeterminate forms $\mathbf{1^\infty, 0^0,}$ and $\mathbf{\infty^0}$

Limits that lead to the indeterminate forms $1^\infty, 0^0, and\ \infty^0$ can sometimes be handled by taking logarithms first, then

$$\lim_{x\to a} ln f(x) = L \qquad \Rightarrow \qquad \lim_{x\to a} e^{ln f(x)} = e^L$$

Example #2: Find each of the following limits.

a) $\lim_{x\to 0}(1+x)^{1/x} =$

b) $\lim_{x\to 1}(x^2-2x+1)^{x-1} =$

c) $\lim_{x\to\infty} x^{1/x} =$

Problem Set #2: Find each of the following limits.

11) $\lim_{x\to 0}(e^x + x)^{1/x} =$	12) $\lim_{x\to 0^+}(2x)^{x/4} =$
13) $\lim_{x\to 1^+}(x-1)^{lnx} =$	14) $\lim_{x\to\infty}(lnx)^{1/x} =$

15) $\lim_{x\to 1} x^{1/(1-x)} =$	16) $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x} =$
17) $\lim_{x\to\frac{\pi}{2}^-} (sinx)^{tanx} =$	18) $\lim_{x\to\infty}(e^{x}+1)^{-2/x} =$
19) $\lim_{x\to 0}(1+2x)^{cscx} =$	20) $\lim_{x\to\infty}(1+2x)^{1/2lnx} =$

Answer Key:
Problem Set #1
1. -1/12 2. 0 3. 1 4. 0 5. 2 6. 0 7. $-\infty$ 8. 5/7 9. 0 10. -1
Problem Set #2
11. e^2 12. 1 13. 1 14. 1 15. 1/e 16. e^3 17. 1 18. e^{-2} 19. e^2 20. $\sqrt{e}$