*Unit #11:* Related Rates
*Topic:* Introduction to Related Rates
*Objective: SWBAT apply derivatives to real life applications.*

# Warm Up #1:

Write each of the following statements mathematically as a rate of change with respect to time.

(a) Max is growing at the rate of 2 inches per year.

(b) My car is losing its resale value at ten dollars per day.

(c) The radius of a circle gets larger by 4 feet each hour.

(d) The outside temperature is dropping at 5°F per minute.

# What are Related Rates?

Calculus is the ________________________________________. Related rate problems are an

important application of calculus that involve finding the ________________ at which some

variable changes over ________________.

We will need to take a formula that relates static variables and use differentiation methods to create one that relates their rates of change.

## *For Example:*

Assume that the radius, $r$, of a sphere is a differentiable function of $t$ and let $V$ be the volume of the sphere. Find an equation that relates $\frac{dV}{dt}$ *and* $\frac{dr}{dt}$. ( $V = \frac{4}{3}\pi r^3$ )

***Steps for Related Rates Problems:***

| |
|---|
| 1) |
| 2) |
| 3) |
| 4) |
| 5) |
| 6) |

## ***Perimeter and Area Problems***

*Example #1:*

The length of a rectangle is decreasing by 2 inches per second and the width is increasing by 3 inches per second. When the length is 10 inches and the width is 6 inches, how fast is the (a) perimeter and (b) area changing?

*Example #2:*

A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius, $r$, of the outer ripple is increasing at a constant rate of 2 feet per second. When the circumference of the ripple is $8\pi$ feet, what rate is the total area, $A$, of the disturbed water increasing?

***Problem Set #1:*** *Read each question carefully and show ALL work.*

1) The side of a square is increasing at a rate of 3.5 feet/minute. At the instant the side has a length of 2 feet, find the following:

   a) How fast is the area of the square changing?
   b) How fast is the perimeter of the square changing?

2) A hypothetical square grows at a rate of 16 $\text{m}^2/\text{min}$. How fast are the sides of the square increasing when the sides are 15 m each?

3) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of $9\pi\ m^2/min$. How fast is the radius of the spill increasing when the radius is 10 $m$?

4) The circumference of a circle is increasing at a rate of 18.6 ft/sec. at the instant the radius has a length of 16.5 feet. How fast is the radius of the circle changing?

5) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 9 cm/min. How fast is the area of the pool increasing when the radius is 12 cm?

6) A snowball melts so that its surface area is decreasing at a rate of 6.4 $cm^2$/min. When the diameter is 6.5 cm, find the rate of change of the radius of the snowball. ($Surface\ Area_{sphere} = 4\pi r^2$)

7) Water slowly evaporates from a circular shaped puddle. The area of the puddle decreases at a rate of $16\pi$ $in^2$/hr. Assuming the puddle retains its circular shape, at what rate is the radius of the puddle changing when the radius is 12 in?

8) The length and width of a rectangle are decreasing at a rate of 1.5 ft/min and increasing at a rate of 1.5 ft/min respectively. At the instant when the length is 14 feet and the width is 20 feet, find the following:

a) The rate at which the area is changing.
b) The rate at which the perimeter is changing.

***Answer Key:***

| | |
|---|---|
| 1) a) $14\ ft^2/min$ <br> b) $14\ ft/min$ | 2) $\frac{8}{15}\ m/min$ |
| 3) $\frac{9}{20}\ m/min$ | 4) $2.960\ ft/sec$ |
| 5) $216\pi\ cm^2/min$ | 6) $-0.078\ cm/min$ |
| 7) $-\frac{2}{3}\ in/hr$ | 8) a) $-9\ ft^2/min$ <br> b) $0\ ft/min$ |