Unit #11: Related Rates
Topic: Introduction to Related Rates
Objective: SWBAT apply derivatives to real life applications.

Warm Up #1:

Write each of the following statements mathematically as a rate of change with respect to time.

(a) Max is growing at the rate of 2 inches per year.

(b) My car is losing its resale value at ten dollars per day.

(c) The radius of a circle gets larger by 4 feet each hour.

(d) The outside temperature is dropping at 5°F per minute.

What are Related Rates?

Calculus is the ____. Related rate problems are an important application of calculus that involve finding the ________________ at which some variable changes over ________________.

We will need to take a formula that relates static variables and use differentiation methods to create one that relates their rates of change.

For Example:

Assume that the radius, r, of a sphere is a differentiable function of t and let V be the volume of the sphere. Find an equation that relates $\frac{dV}{dt}$ *and* $\frac{dr}{dt}$. ($V = \frac{4}{3}\pi r^3$)

Steps for Related Rates Problems:

1)
2)
3)
4)
5)
6)

Perimeter and Area Problems

Example #1:

The length of a rectangle is decreasing by 2 inches per second and the width is increasing by 3 inches per second. When the length is 10 inches and the width is 6 inches, how fast is the (a) perimeter and (b) area changing?

Example #2:

A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius, r, of the outer ripple is increasing at a constant rate of 2 feet per second. When the circumference of the ripple is 8π feet, what rate is the total area, A, of the disturbed water increasing?

Problem Set #1: *Read each question carefully and show ALL work.*

1) The side of a square is increasing at a rate of 3.5 feet/minute. At the instant the side has a length of 2 feet, find the following:

   a) How fast is the area of the square changing?
   b) How fast is the perimeter of the square changing?

2) A hypothetical square grows at a rate of 16 m^2/min. How fast are the sides of the square increasing when the sides are 15 m each?

3) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of $9\pi\ m^2/min$. How fast is the radius of the spill increasing when the radius is 10 m?

4) The circumference of a circle is increasing at a rate of 18.6 ft/sec. at the instant the radius has a length of 16.5 feet. How fast is the radius of the circle changing?

5) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 9 cm/min. How fast is the area of the pool increasing when the radius is 12 cm?

6) A snowball melts so that its surface area is decreasing at a rate of 6.4 cm^2/min. When the diameter is 6.5 cm, find the rate of change of the radius of the snowball. $(Surface\ Area_{sphere} = 4\pi r^2)$

7) Water slowly evaporates from a circular shaped puddle. The area of the puddle decreases at a rate of 16π in^2/hr. Assuming the puddle retains its circular shape, at what rate is the radius of the puddle changing when the radius is 12 in?

8) The length and width of a rectangle are decreasing at a rate of 1.5 ft/min and increasing at a rate of 1.5 ft/min respectively. At the instant when the length is 14 feet and the width is 20 feet, find the following:

   a) The rate at which the area is changing.
   b) The rate at which the perimeter is changing.

Answer Key:

1) a) $14\ ft^2/min$ b) $14\ ft/min$	2) $\frac{8}{15}\ m/min$
3) $\frac{9}{20}\ m/min$	4) $2.960\ ft/sec$
5) $216\pi\ cm^2/min$	6) $-0.078\ cm/min$
7) $-\frac{2}{3}\ in/hr$	8) a) $-9\ ft^2/min$ b) $0\ ft/min$