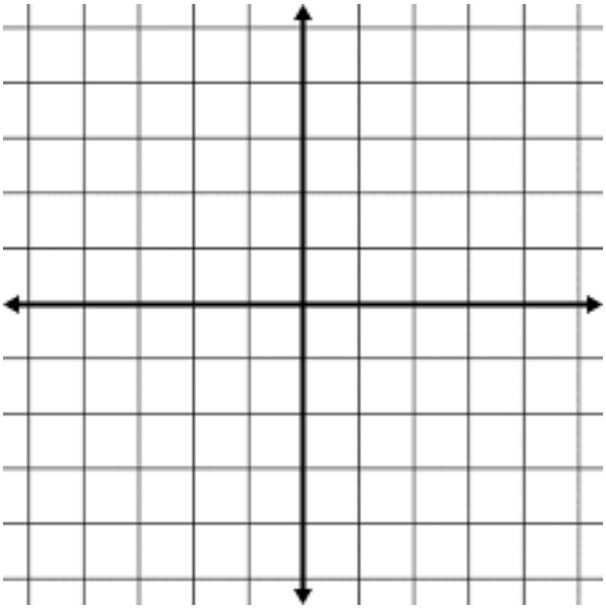
Unit #10: Applications of Differentiation
Topic: Mean Value Theorem
Objective: SWBAT explain whether or not a function satisfies the MVT.
SWBAT find a value of c that satisfies the MVT on a given interval.

Warm Up #1:

Given: $f(x) = \frac{x^2 - 9}{3x}$

a) Graph $f(x)$ over the interval $[1, 4]$.

x	$f(x)$
1	
2	
3	
4	



b) Find the average rate of change of f over $[1, 4]$.

c) Find the instantaneous rate of change of f at $x = 2$.

What do you notice??

The Mean Value Theorem (MVT):

The mean value theorem connects the average rate of change of a function over an interval to the instantaneous rate of change of the function at a point within the same interval.

The theorem states:

If $f(x)$ is _____ at EVERY point on the _____ interval [a,b] and _____ at EVERY point on the _____ interval (a,b), then there is at least one point c in (a,b) at which

Example #1: Find all values of c that satisfies the conclusions of the Mean Value Theorem for $f(x) = \sqrt{2-x}$ on $[-7, 2]$.

Can we do the dame thing to find a value of c that satisfies the MVT for $f(x)$ on $[-5, 4]$? Explain your reasoning.

What goes up MUST come down!!

Rolle's Theorem:

Rolle's theorem is a special case of the MVT that states:

If $f(x)$ is continuosu at EVERY point on the closed interval [a,b] and differentiable at EVERY point on the open interval (a,b), AND _____, then there is at least one value of c where _____.

Example #2: Find all values of c that satisfies the conclusions of Rolle's Theorem for $y = -x^2 + 8x - 17$ on $[2, 6]$.

Practice Problems: For each problem, determine if the Mean Value Theorem or Rolle's Theorem can be applied. If it can, find all values of c that satisfy the theorem. If it cannot, explain why not.

1) $y = x^2 + \frac{1}{2}$; $[1, 2]$	2) $f(x) = sin2x$; $[0, \pi]$

3) $g(x) = \frac{1}{x-2}$; $[1,3]$	4) $f(x) = -\frac{x^2}{2} + x - \frac{1}{2}$; $[-2,1]$
5) $y = \frac{x^2}{2} - 2x - 1$; $[-1,1]$	6) $h(x) = -(-2x+6)^{1/2}$; $[-2,3]$
7) $y = \frac{x^2-1}{x}$; $[-1,1]$	8) $f(x) = -x^3 + 4x^2 - 3$; $[0,4]$

9) $f(x) = -(6x + 24)^{2/3}$; $[-4, -1]$	10) $y = \frac{-x^2 + 9}{4x}$; $[1, 3]$
11) $g(x) = 5cosx$; $[0, 2\pi]$	12) $y = -\frac{x^2}{4x + 8}$; $[-3, -1]$
13) $f(x) = \frac{x + 1}{x}$; $\left[\frac{1}{2}, 2\right]$	14) $y = 5 - \frac{4}{x}$; $[1, 4]$

15) $y = \frac{1}{x-1}$; $[2,4]$	16) $g(x) = (x-3)^{2/3}$; $[1,4]$

Answer Key

1) MVT ; $x = \frac{3}{2}$	2) Rolles ; $x = \frac{\pi}{4}$, $\frac{3\pi}{4}$	3) neither ; not continuous at $x = 2$.
4) MVT ; $x = -\frac{1}{2}$	5) MVT ; $x = 0$	6) MVT ; $x = \frac{7}{4}$
7) neither ; not diff. at $x = 0$.	8) Rolles ; $x = \frac{8}{3}$	9) MVT ; $x = -\frac{28}{9}$
10) MVT ; $x = \sqrt{3}$	11) Rolles ; $x = 0,\ \pi, 2\pi$	12) neither ; not continuous at $x = -2$.
13) MVT ; $x = \sqrt{\frac{2}{3}}$	14) MVT ; $x = 2$	15) MVT ; $x = \frac{2}{ln3} + 1$
16) neither ; not diff. At $x = 3$.		