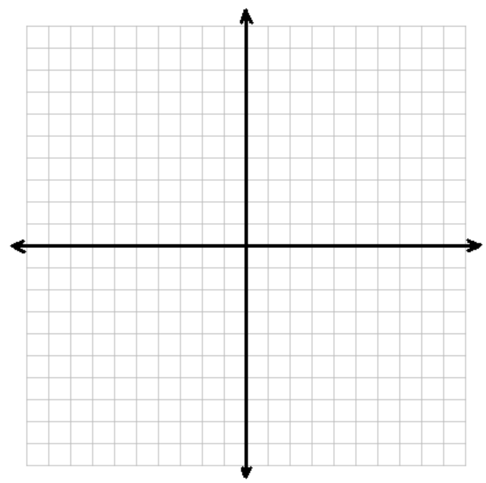
Unit #6: Parametric and Polar Derivatives
Topic: Parametric Derivatives
Objective: SWBAT find the first and second derivatives of a parametric equation.

Warm Up #1:

Find and graph a Cartesian equation for a curve that contains the parametrized curve given below.

$$x = 3 - 3t \;\; and \;\; y = 2t \quad from\; 0 \leq t \leq 1$$



Slope and Concavity of Parametric Equations

There are times when we need to describe motion (or a curve) that is not a function. We can do this by writing equations for the x *and* y coordinates in terms of a third variable, usually t or θ. These are called parametric equations, where "t" is the parameter.

$$x = f(t) \quad y = g(t)$$

We can analyze the slope and concavity of parametric curves the same way we can with explicitly defined curves.

Parametric Differentiation Formulas:

$$\frac{dy}{dx} =$$

$$\frac{d^2y}{dx^2} =$$

Example #1: Given the following curve defined parametrically by $x = \ln(5t)$ and $y = e^{5t}$.

a) Find $\frac{dy}{dx}$.

b) Find $\frac{d^2y}{dx^2}$.

Example #2: Consider the curve defined parametrically by $x = t^2 - 3t$ and $y = t^3$.

a) Find the values of t where the tangent line to the curve is (1) horizontal or (2) vertical.

b) Find the equation of the line tangent to the curve at $t = 1$.

c) Find all points of inflection on the curve. Justify your answer.

Problem Set #1:

1. Find the tangent line(s) to the parametric curve given by $x = t^5 - 4t^3$ and $y = t^2$ at $(0,4)$.

2. Determine the $x - y$ coordinates of the points where the following parametric equations will have horizontal or vertical tangents.

$$x = t^3 - 3t \qquad y = 3t^2 - 9$$

3. Determine the values of t for which the parametric curve given by the following set of parametric equations is concave up and concave down.

$$x = 1 - t^2 \qquad y = t^7 + t^5$$

4. An object moves along the path $x = 3t$ and $y = \cos(2t)$, where t is time.

a) Write the equation for the line tangent to the curve at $t = \frac{\pi}{3}$.

b) Find the smallest value of t for which the y-coordinate is a local maximum.

c) Find $\frac{d^2y}{dx^2}$ when $t = 2$. What does this tell you about the concavity of the graph at $t = 2$?

5. Given the parametrized curve $x = 2cost$ and $y = 3sint, 0 \leq t \leq 2\pi$.

a) Find an equation for the tangent at the point where $t = \frac{3\pi}{4}$.

b) Find an equation for the normal at the point where $t = \frac{3\pi}{4}$.

6. A curve C is defined by the parametric equations $x = t^2 + t - 1,\ y = t^3 - t^2$.

a) Find $\frac{dy}{dx}$ in terms of t.

b) Find an equation of the tangent line to C at the point where $t = 2$.

Answer Key

1. $y - 4 = \frac{1}{8}x \quad and \quad y - 4 = -\frac{1}{8}x$

2. horizontal at (0,-9); vertical at (2,-6) and (-2,-6)

3. concave down $t < 0$ and concave up $t > 0$

4. a) $y + \frac{1}{2} = -\frac{\sqrt{3}}{3}(x - \pi)$

   b) $t = \pi$

   c) concave up

5. a) $y - \frac{3\sqrt{2}}{2} = \frac{3}{2}(x + \sqrt{2})$

   b) $y - \frac{3\sqrt{2}}{2} = -\frac{2}{3}(x + \sqrt{2})$

6. a) $\frac{dy}{dx} = \frac{3t^2 - 2t}{2t + 1}$

   b) $y - 4 = \frac{8}{5}(x - 5)$