

Unit #3: Differential Equations

Topic: Separable Differential Equations

Objective: *SWBAT find a general solution to a separable differential equation.*

Warm Up #1:

Find the particular solution to the equation $\frac{d^2y}{dx^2} = 2x + 1$ when $x = 0$,

$\frac{dy}{dx} = -1$, and $y = 3$.

A differential equation is an equation which contains a derivative such as $\frac{dy}{dx}$. A differential equation can be solved by finding an expression for y in terms of x without the derivative.

What if there is more than one variable?



A differential equation of the form $\frac{dy}{dx} = f(y)g(x)$ is called a **separable differential equation** if it can be solved by **separating the variables** and then antidifferentiating each side with respect to the given variable.

Example #1: Find the general solution for the following equation $\frac{dy}{dx} = \frac{(x^2+1)}{2y}$

Example #2: Solve the equation $\frac{dy}{dx} = e^{x-y}$ to find the general solution.

Problem Set #1: Find the general solution to each of the following differential equations.

1) $\frac{dy}{dx} = 4x^3y^2$

2) $\frac{dy}{dx} = \frac{1}{x^3y^2}$

$$3) \frac{dy}{dx} = \frac{\sqrt{x}}{e^y}$$

$$4) \frac{dy}{dx} = \frac{y^2(x-3)}{x^3}$$

$$5) \frac{dy}{dx} = \frac{x}{y}$$

$$6) \frac{dy}{dx} = 9x^2y$$

$$7) \frac{dy}{dx} = 3x^2e^{-y}$$

$$8) \frac{dy}{dx} = \frac{y}{x(x+1)}$$

9) $\frac{dy}{dx} = \frac{xy}{x^2+1}$

10) $\frac{dy}{dx} = e^{2x+y}$

11) $\frac{dy}{dx} = xy + 2y$

12) $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$

Answer Key

1. $y = -\frac{1}{x^4 + C}$

2. $y = (-\frac{3}{2}x^2 + C)^{1/3}$

3. $y = \ln \left| \frac{2}{3}x^{3/2} + C \right|$

4. $y = \frac{2x^2}{2x - 3 - 2x^2C}$

5. $y = \sqrt{x^2 + C}$

6. $y = e^{3x^3 + C}$

7. $y = \ln|x^3 + C|$

8. $y = e^c \left| \frac{x}{x+1} \right|$

9. $y = e^c \sqrt{x^2 + 1}$

10. $y = -\ln \left| -\frac{1}{2}e^{2x} - C \right|$

11. $y = e^{\frac{1}{2}x^2 + 2x + C}$

12. $y = e^{\sqrt{x} + C}$