

Unit #9: Rates of Change and Particle Motion

Topic: The Derivative and Rates of Change

Objective: SWBAT will be able to interpret the derivative in order to model and explain rates of change.

Warm Up #1:

The height of a ball thrown upward at a speed of 30 ft/s from a height of 15 feet after t seconds is given by:

$$S(t) = 15 + 30t - 16t^2$$

(a) Find the average velocity of the ball in the first 2 seconds after it is thrown.

(b) Find the instantaneous velocity of the ball when $t = 2$ seconds.

Explain, in your own words, the meaning of the average rate of change vs. the instantaneous rate of change.

The Derivative and Rates of Change

If $f(x)$ represents a quantity, then $f'(x)$ represents the _____
_____ of that function.

The derivative gives us a way to talk about how fast ANYTHING is changing at _____
_____, but $f(x)$ can represent any quantity such as particle position, velocity, area of a circle, outdoor temperature, amount of rainfall, cost of production, etc.

NOTE: The word **instantaneous** is often used, even when x does not represent time, but the word is also often omitted

The units of the derivative of any function $y = f(x)$ will always be _____
over _____.

Example #1:

The temperature T , in degrees Fahrenheit, of a cold yam placed in a hot oven is given by $T = f(t)$, where t is the time in minutes since the yam was put in the oven.

- (a) In a complete sentence with units, translate the practical meaning of the equation $f(20) = 255$.
- (b) What is the sign of $f'(t)$? Why?

- (c) What are the units of $f'(20)$?
- (d) In a complete sentence with units, what is the practical meaning of the statement $f'(20) = 2$?

Example #2:

An ice cream company knows that the cost, C (in dollars), to produce q quarts of cookie dough ice cream is a function of the number of quarts they produce, so $C = f(q)$.

- (a) If $f(200) = 70$, explain in a full sentence with units exactly what this mathematical “sentence” is telling us in the context of the problem.
- (b) If $f'(200) = 3$, explain in a full sentence with units exactly what this mathematical “sentence” is telling us in the context of the problem.
- (c) If $f''(200) = -0.1$ explain in a full sentence with units exactly what this mathematical “sentence” is telling us in the context of the problem.

Example #3:

- (a) Find the rate of change of the area A of a circle with respect to its radius r .
- (b) Evaluate the rate of change of A at $r = 5$ and at $r = 10$.
- (c) If r is measured in inches and A is measured in square inches, what units, would be appropriate for $\frac{dA}{dr}$?

Problem Set #1:

- 1) If $g(v)$ is the fuel efficiency, in miles per gallon, of a car going at v miles per hour,
- (a) What are the units of $g'(v)$?
- (b) In a complete sentence with units, what is the practical meaning of the statement $g'(55) = -0.54$?

2) An economist is interested in how the price of a graphing calculator affects its sales. Suppose that at a price of p dollars, a quantity, q , calculators are sold, then the quantity of calculators sold is a function of the calculator's price, that is, if $q = f(p)$.

(a) In a complete sentence using correct units, explain the meaning of $f(150) = 20,000$.

(b) In a complete sentence using correct units, explain the meaning of $f'(150) = -50$

(c) Assuming the rate from part (b) holds for $150 \leq p \leq 170$, how many calculators are predicted to sell when the price of a calculator is \$160? Show the work that leads to your answer.

3) Suppose that $C(T)$ is the cost of heating your house, in dollars per day, when the temperature outside is T degrees Fahrenheit.

(a) Explain in practical terms the meaning of $C'(23) = -0.21$.

(b) If $C(23) = 2.94$ and $C'(23) = -0.21$, approximate the cost to heat your house when the temperature is 20 degrees, $C(20)$. Show the work that leads to your answer.

