Unit 6: Continuity
Topic: Continuity on an Interval
Objective: SWBAT determine if a function is continuous on a given interval.
SWBAT use the Intermediate Value Theorem to show the existence of certain values on an interval.

Warm Up #2:

Let f be defined by the function $f(t) = \dfrac{4t + 12}{t^2 - 2t - 15}$.

Which of the following statement(s) is(are) true? Explain your reasoning.

I. f has a limit at $t = -3$

II. f is continuous at $t = 5$

III. f has a vertical asymptotes at $t = 5 \text{ and } t = -3$

IV. $f(-3)$ exists

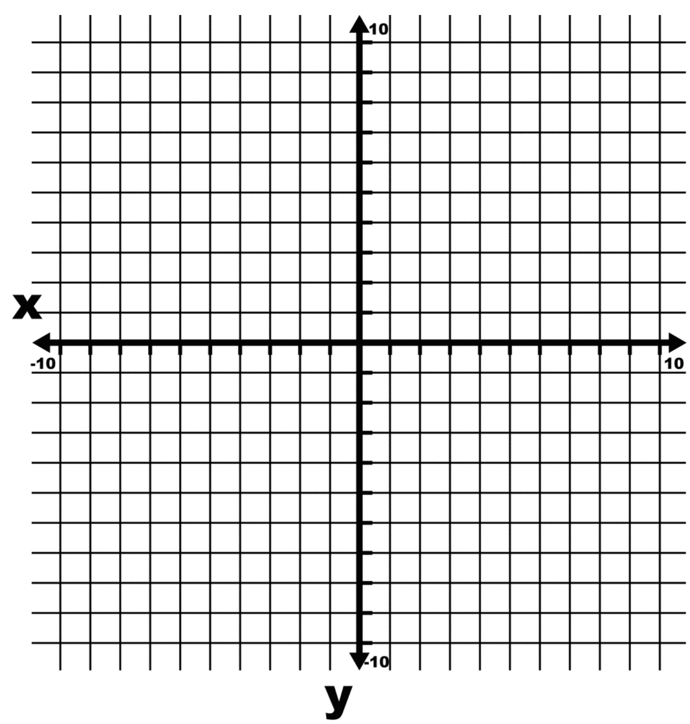
Continuity on an Interval

A function is **continuous** on an interval if and only if the function $f(x)$ is continuous at **every** point in the interval.

Let's investigate this further:

Draw the graph of the function

$$f(x) = \begin{cases} |x + 2| - 1, & for\ x \le 1 \\ 4, & for\ 1 < x < 5 \\ 9 - x, & for\ 5 \le x \le 9 \end{cases}$$



On what interval(s) is the function above continuous?

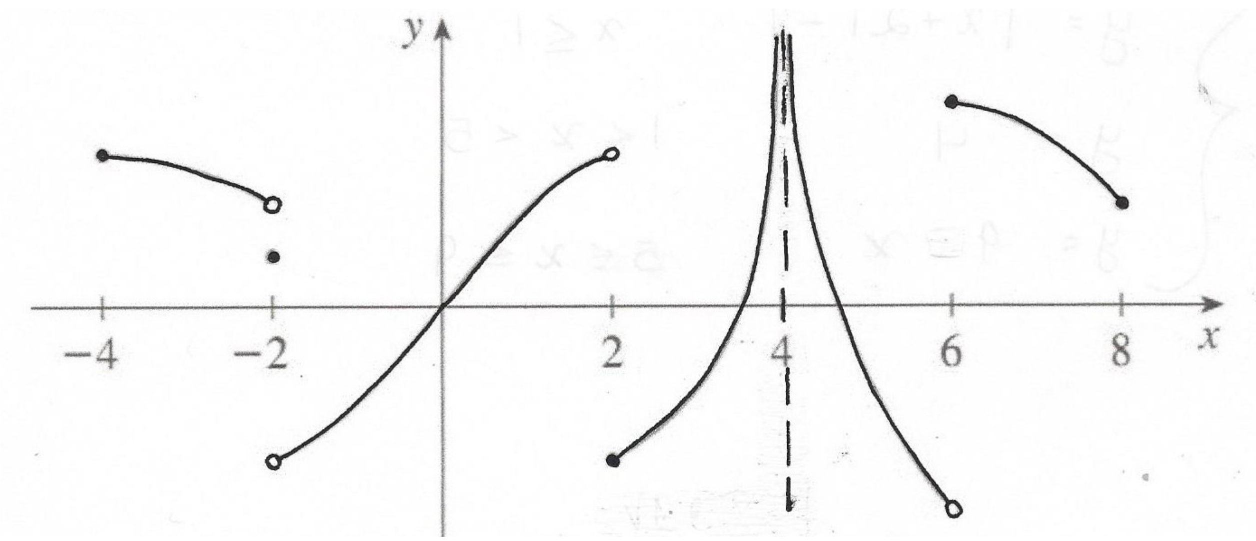
So, more specifically,

A function f is continuous on an ______________________ , (a, b), if it is continuous at every number of the interval.

A function f is continuous on a ______________________ , $[a, b]$, if it is continuous on the open interval and

Now Let's Try:

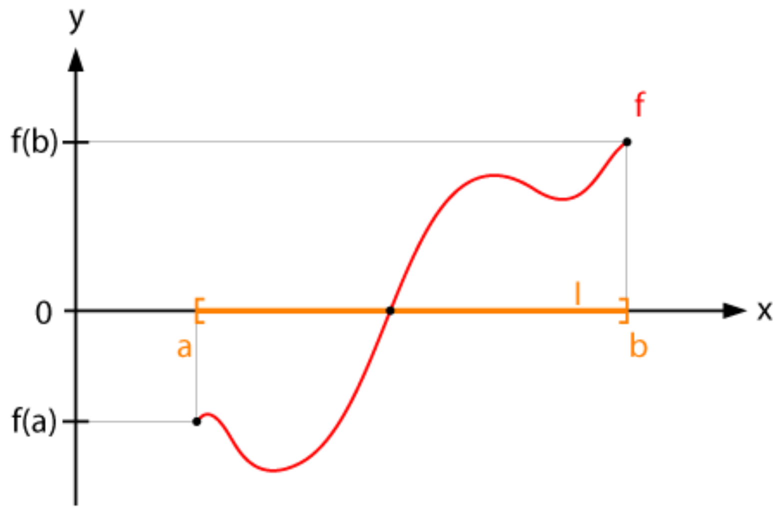
Determine if the function given below is continuous on each of the following intervals. If not, rewrite the interval so that the function in continuous in the interval.



1) $[-4, 2)$ 2) $[-2, 2]$ 3) $(2, 4)$ 4) $(4, 8)$ 5) $[6, 8]$

The Intermediate Value Theorem

If f is a function that is continuous over the closed interval $[a, b]$ and y_0 is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that $f(c) = y_0$.



The Intermediate Value Theorem is an existence theorem. It tells us under what conditions c exists, but nothing about the actual value of c.

Examples:

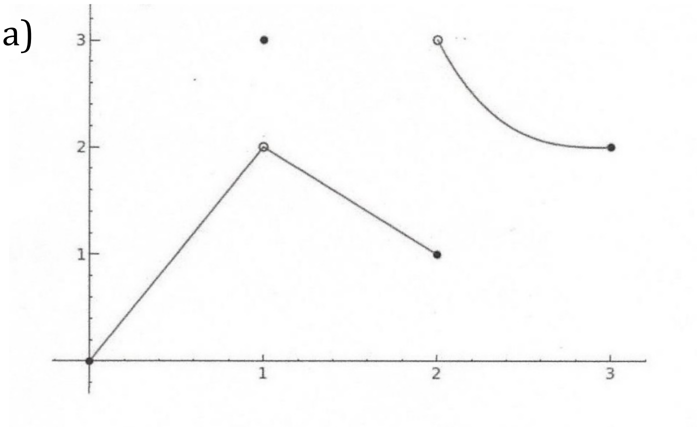
1) Given $f(x) = \frac{2x-3}{x-5}$, show that $f(x) = -3$ for some value on $[0,4]$.

2) Given $f(x) = 3x^2 - 2x - 5$, show that $f(x) = 0$ for some value on $[1,2]$.

Problem Set #2:

1) Determine the intervals on which each of the following functions is continuous.

a)



b) $f(x) = \begin{cases} 1 + x^2, & x \le 0 \\ 2 - x, & 0 < x \le 2 \\ (x-2)^2, & x > 2 \end{cases}$

c) $g(x) = \begin{cases} x + 1, & x \le 1 \\ \frac{1}{x}, & 1 < x < 3 \\ \sqrt{x-3}, & x \ge 3 \end{cases}$

2) Does the IVT hold for each of the following functions on the given interval? Why/Why not?

a) $f(x) = 2 + x - x^2$; $[-1,4]$

b) $g(x) = \frac{x^2+2x-8}{x^2+x-6}$; $[0,3]$

c) $h(x) = \frac{1}{x^2}$; $[-2,3]$

d) $\frac{1}{3x}$; $[2,5]$

3) Show whether the conditions of the IVT hold for the given value of c. Explain your answer.

a) $f(x) = x^2 + x - 1$; $[0,5]$; $f(c) = 11$

b) $g(x) = x^3 - x^2 + x - 2$; $[0,1]$; $f(c) = 0$

c) $f(x) = \frac{x^2 - x}{x - 1}$; $\left[\frac{5}{2}, 5\right]$; $f(c) = 6$

d) $h(x) = 2cost - 3t$; $[0,1]$; $f(c) = 0$

e) $f(x) = \sqrt{2x - 1}$; $[1,9]$; $f(c) = 3$