Unit #5: Limits
Topic: Finding Limits Graphically
Objective: SWBAT find the limit of a function by using the graph and by direct substitution.

Warm Up #2:

CALCULATOR ALLOWED

1. Consider the graph of the function: $f(x) = \frac{x^2-5x+4}{x-4}$

   a) Are there any values of x for which the function is undefined? Why?

   b) Enter the function into your graphing calculator and using the table set begin your table at x =3 and set the $\Delta Table$ to 0.1. Use the values given to fill in the table below.

x	3.5	3.6	3.7	3.8	3.9	4	4.1	4.2	4.3	4.4	4.5
y											

   c) Explain in your own words what is happening at $x = 4$.

2. Consider the graph of the function: $f(x) = \frac{\sqrt{x}-1}{x-1}$

   a) Are there any values of x for which the function is undefined? Why?

   b) Enter the function into your graphing calculator and using the table set begin your table at $x = 0$ and set the $\Delta Table$ to 0.1. Use the values given to fill in the table below.

x	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
y											

   c) Explain in your own words what is happening at $x = 1$.

What is the Limit of a Function?

In mathematics, the *limit of a function* is a fundamental concept in calculus.
A limit is a certain value to which a function approaches. Finding a limit means finding what value *y* is as *x* approaches a certain number.

$$\boxed{\lim_{x \to c} f(x) = L.}$$

We are only concerned about what happens with *f(x)* as *x* gets arbitrarily close to *c*.
We don't actually care about what happens when *x=c.*

How do we find Limits?

There are several methods that can be used to find limits.

1. *Graphically*

We can approach a given value from the left, which will give us the left-sided limit, or from the right, which will give us the right-sided limit.

The limit exists if the **left-sided limit = the right-sided limit**.

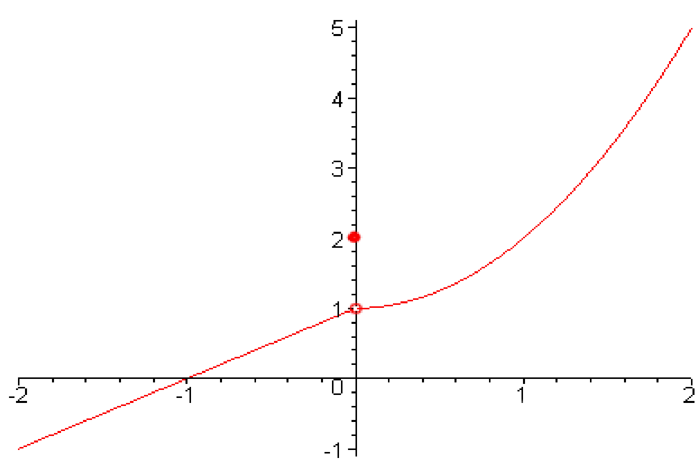
Example #1: Find

a) $\lim_{x \to 0^-} f(x)$

b) $\lim_{x \to 0^+} f(x)$

c) $\lim_{x \to 0} f(x)$

d) $f(0)$



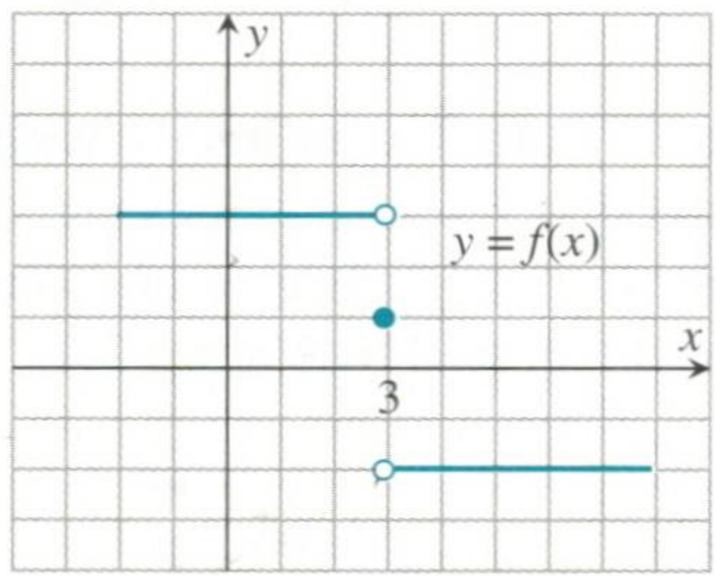
Example #2: Find

a) $\lim_{x\to 3^-} f(x)$

b) $\lim_{x\to 3^+} f(x)$

c) $\lim_{x\to 3} f(x)$

d) $f(3)$



2. *By Direct Substitution*

We can find the limits of polynomial functions and *most* rational functions by using direct substitution.

Example #3: Find the limits for each of the following.

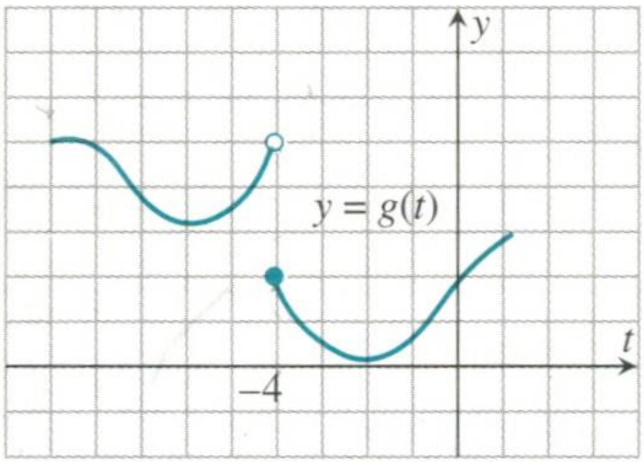
a) $\lim_{x\to 3}(2x+4)$

b) $\lim_{x\to -1/2} 3x^2(2x-1)$

c) $\lim_{x\to 2} \frac{x^2+5x+6}{x+2}$

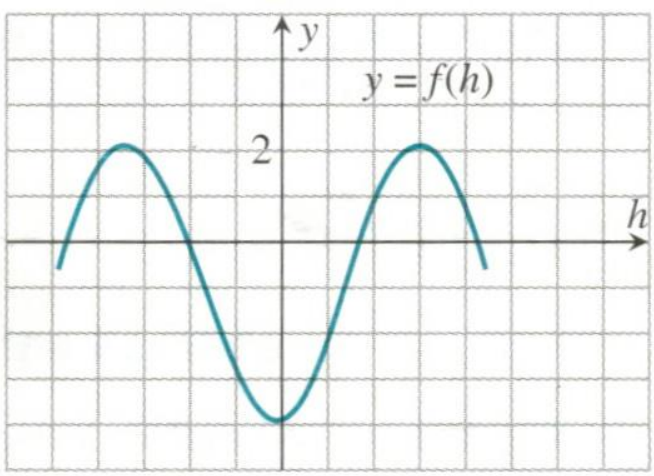
Problem Set #2:

1.



(a) $\lim_{t \to -4^-} g(t)$ **(b)** $\lim_{t \to -4^+} g(t)$ **(c)** $\lim_{t \to -4} g(t)$ **(d)** $g(-4)$

2.

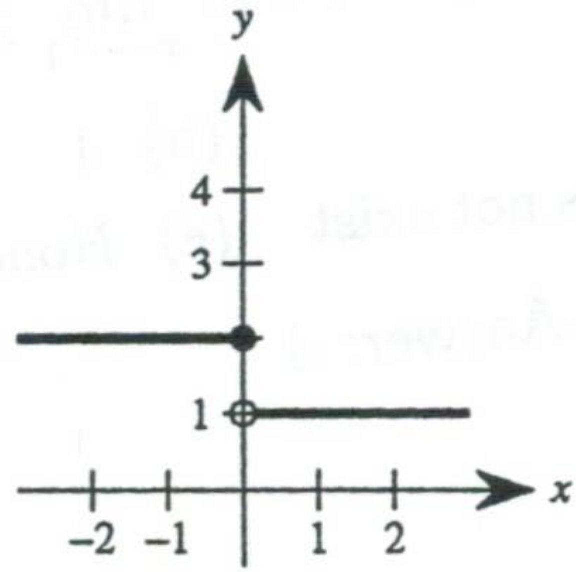


(a) $\lim_{h \to 0^-} f(h)$ **(b)** $\lim_{h \to 0^+} f(h)$ **(c)** $\lim_{h \to 0} f(h)$ **(d)** $f(0)$

3. Find the limit for each of the following:

a) $\lim_{x \to 2} \sqrt{x+3}$ b) $\lim_{x \to -3} \frac{x^2+4x+3}{x^2-3}$ c) $\lim_{x \to 0} e^x cosx$

4. Find



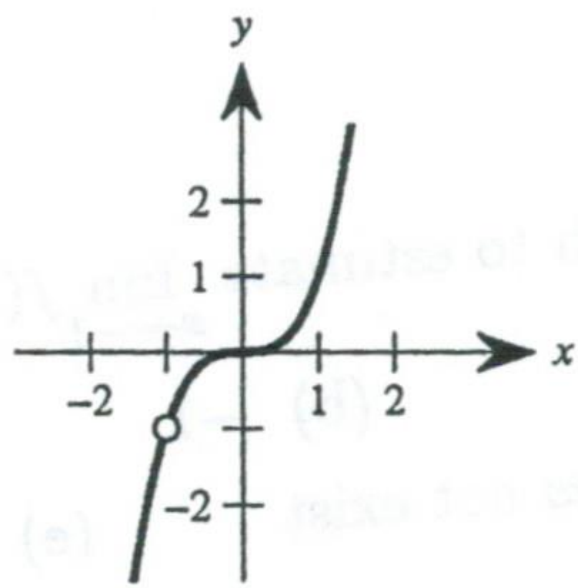
a) $\lim_{x \to 0^-} g(x)$

b) $\lim_{x \to 0^+} g(x)$

c) $\lim_{x \to 0} g(x)$

d) $g(0)$

5. Find



a) $\lim_{x \to -1^-} h(x)$

b) $\lim_{x \to -1^+} h(x)$

c) $\lim_{x \to -1} h(x)$

d) $h(-1)$

6. Find the limit for each of the following:

a) $\lim_{a \to 0.2} (3a + 4)$

b) $\lim_{x \to -1} (3x^4 - 2x^3 + 4x)$

c) $\lim_{t \to 4} \frac{3t-14}{t+1}$

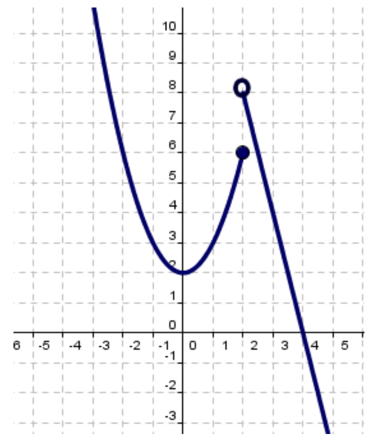
Name ______________________________________ **Date** ______________

Homework - The Limit of a Function

1. Find

a) $\lim\limits_{x \to 2^-} f(x)$ b) $\lim\limits_{x \to 2^+} f(x)$

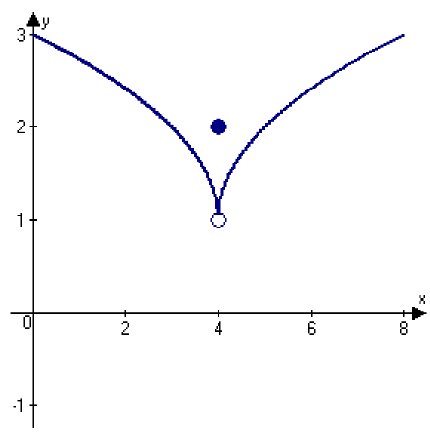
c) $\lim\limits_{x \to 2} f(x)$ d) $f(2)$



2. Find

a) $\lim\limits_{x \to 4^-} g(x)$ b) $\lim\limits_{x \to 4^+} g(x)$

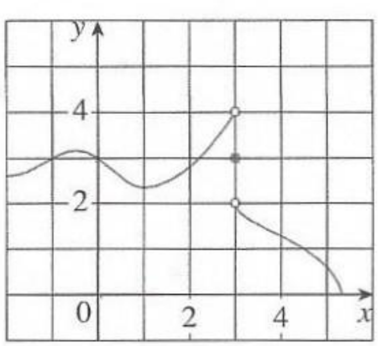
c) $\lim\limits_{x \to 4} g(x)$ d) $g(4)$



3. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim\limits_{x \to 0} f(x)$ (b) $\lim\limits_{x \to 3^-} f(x)$ (c) $\lim\limits_{x \to 3^+} f(x)$

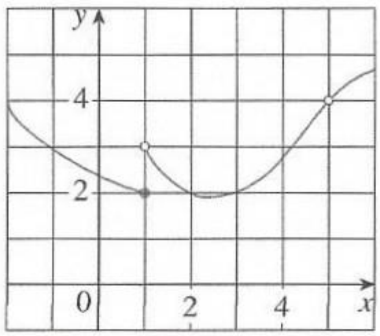
(d) $\lim\limits_{x \to 3} f(x)$ (e) $f(3)$



4. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 1^-} f(x)$ (b) $\lim_{x \to 1^+} f(x)$ (c) $\lim_{x \to 1} f(x)$

(d) $\lim_{x \to 5} f(x)$ (e) $f(5)$



5. Find the limit for each of the following using direct substitution.

a) $\lim_{x \to 0}(4x^2 + 2x + 5)$ b) $\lim_{b \to 2} \frac{1}{b^2-1}$ c) $\lim_{x \to 0} 3cosx$

6. For the function g whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{t \to 0^-} g(t)$ (b) $\lim_{t \to 0^+} g(t)$ (c) $\lim_{t \to 0} g(t)$

(d) $\lim_{t \to 2^-} g(t)$ (e) $\lim_{t \to 2^+} g(t)$ (f) $\lim_{t \to 2} g(t)$

(g) $g(2)$ (h) $\lim_{t \to 4} g(t)$