

Unit #2: Methods of Integration

Topic: More U-Substitution

Objective: *SWBAT find an indefinite integral of a composite function by using a u-substitution.*

## Warm Up #2:

Evaluate each of the following:

a)  $\int \frac{2x}{\sqrt{4-4x^2}} dx$

b)  $\int \frac{2}{\sqrt{4-4x^2}} dx$

### *Tips & Guidelines*

Sometimes we need to look harder to find the appropriate substitution.

- If something is being raised to an exponent, that will be  $u$ .
- If one function is 1 degree higher than the other function, that will be  $u$ .
- If  $e$  is being raised to an exponent, that exponent will be  $u$ .
- If you have one trig function, the inside function will be  $u$ .
- If something is under a radical, that will be  $u$ .



*Example #1:* Evaluate  $\int \sqrt{x}\sqrt{x\sqrt{x} + 1} dx$

*Example #2:* Evaluate  $\int \frac{e^{\ln\sqrt{x}}}{4x} dx$

*Problem Set #2: Evaluate each of the following integrals using an appropriate substitution.*

$$1) \int (x^3 - 1)^9 x^2 dx =$$

- A)  $\frac{(x^3 - 1)^8}{8} + c$   
 B)  $\frac{x^3(x^3 - 1)^{10}}{10} + c$   
 C)  $\frac{(x^3 - 1)^{10}}{10} + c$   
 D)  $\frac{x^3(x^3 - 1)^{10}}{30} + c$   
 E)  $\frac{(x^3 - 1)^{10}}{30} + c$

$$2) \int x\sqrt{5x^2 - 3} dx =$$

- A)  $\frac{1}{15}(5x^2 - 3)^{\frac{3}{2}} + c$   
 B)  $\frac{x^2(5x^2 - 3)^{\frac{3}{2}}}{3} + c$   
 C)  $\frac{1}{10}(5x^2 - 3)^{\frac{3}{2}} + c$   
 D)  $\frac{2}{3}(5x^2 - 3)^{\frac{3}{2}} + c$   
 E)  $10(5x^2 - 3)^{\frac{3}{2}} + c$

$$3) \int \frac{2x^2}{\sqrt{x^3 + 3}} dx =$$

- A)  $\frac{4}{3}\sqrt{x^3 + 3} + c$   
 B)  $\frac{2}{3}\sqrt{x^3 + 3} + c$   
 C)  $\frac{1}{3}\sqrt{x^3 + 3} + c$   
 D)  $\frac{4}{3\sqrt{x^3 + 3}} + c$   
 E)  $\frac{3}{4}\sqrt{x^3 + 3} + c$

$$4) \int \frac{dx}{(5x + 3)^7} =$$

- A)  $\frac{1}{8(5x + 3)^8} + c$   
 B)  $-\frac{1}{30(5x + 3)^6} + c$   
 C)  $-\frac{1}{40(5x + 3)^8} + c$   
 D)  $-\frac{1}{6(5x + 3)^6} + c$   
 E)  $\frac{1}{30(5x + 3)^6} + c$

$$5) \int \frac{(\sqrt{x} - 1)^5}{\sqrt{x}} dx =$$

- A)  $\frac{(\sqrt{x} - 1)^6}{12} + c$   
 B)  $\frac{(x\sqrt{x} - x)^6}{6} + c$   
 C)  $\frac{(x - \sqrt{x})^6}{6} + c$   
 D)  $\frac{(\sqrt{x} - 1)^6}{3} + c$   
 E)  $\frac{(\sqrt{x} - 1)^6}{6} + c$

$$6) \int (x^2 + 1)(x^3 + 3x - 7)^{\frac{3}{5}} dx =$$

- A)  $\frac{5}{24}(x^3 + 3x - 7)^{\frac{8}{5}} + c$   
 B)  $\frac{5}{8}(x^3 + 3x - 7)^{\frac{8}{5}} + c$   
 C)  $\frac{1}{3}(x^3 + 3x - 7)^{\frac{2}{5}} + c$   
 D)  $\frac{5}{16}(x^2 + 1)^2(x^3 + 3x - 7)^{\frac{8}{5}} + c$   
 E)  $\frac{8}{15}(x^3 + 3x - 7)^{\frac{8}{5}} + c$

$$7) \int \frac{dt}{\sqrt{t}(1-\sqrt{t})^2} =$$

- A)  $-\frac{1}{2(1-\sqrt{t})^3} + c$   
 B)  $-\frac{2}{3(1-\sqrt{t})^3} + c$   
 C)  $\frac{2}{1-\sqrt{t}} + c$   
 D)  $-\frac{2}{1-\sqrt{t}} + c$   
 E)  $\frac{1}{2(1-\sqrt{t})} + c$

$$8) \int \frac{x+2}{(x^2+4x-1)^2} dx =$$

- A)  $-\frac{3}{(x^2+4x-1)^3} + c$   
 B)  $\frac{x^2+2x}{6(x^2+4x-1)^3} + c$   
 C)  $\frac{3}{(x^2+4x-1)^3} + c$   
 D)  $-\frac{1}{2x^2+8x-2} + c$   
 E)  $\frac{1}{2x^2+8x-2} + c$

$$9) \int (x^2+2x+1)^{10} dx =$$

- A)  $\frac{(x+1)^{19}}{19} + c$   
 B)  $\frac{(x+1)^{21}}{21} + c$   
 C)  $\frac{(x+1)^{13}}{13} + c$   
 D)  $\frac{1}{11} \left( \frac{x^3}{3} + x^2 + x \right)^{11} + c$   
 E)  $\frac{(x^2+2x+1)^{11}}{11} + c$

10) If functions  $f$  and  $g$  are differentiable functions, then  $\int g'(f(x)) f'(x) dx =$

- A)  $g'(x) + c$   
 B)  $g(x) + c$   
 C)  $g(x) f(x) + c$   
 D)  $g(f'(x)) + c$   
 E)  $g(f(x)) + c$

$$11) \int x\sqrt{x-1} dx =$$

- A)  $\frac{2}{3}(x^2-x)^{\frac{3}{2}} + c$   
 B)  $\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + c$   
 C)  $\frac{5}{2}(x-1)^{\frac{5}{2}} + \frac{3}{2}(x-1)^{\frac{3}{2}} + c$   
 D)  $\frac{1}{3}x^2(x-1)^{\frac{3}{2}} + c$   
 E)  $\frac{1}{2}(x-1)^4 + c$

$$12) \int x^3 \cos x^4 dx =$$

- A)  $\frac{x^4}{4} \sin x^4 + c$   
 B)  $-\frac{1}{4} \sin x^4 + c$   
 C)  $-\frac{x^4}{4} \sin x^4 + c$   
 D)  $\frac{1}{4} \sin x^4 + c$   
 E)  $\frac{x^4}{4} \sin \frac{x^5}{5} + c$

$$13) \int \sin 5x dx =$$

- A)  $\cos 5x + c$   
 B)  $-5 \cos 5x + c$   
 C)  $-\frac{1}{5} \cos 5x + c$   
 D)  $\frac{1}{5} \cos 5x + c$   
 E)  $5 \cos 5x + c$

$$14) \int (\tan^3 x)(\sec^2 x) dx =$$

- A)  $\frac{1}{4} \tan^4 x + c$   
 B)  $\frac{1}{2} \sec^2 x + c$   
 C)  $\frac{1}{2} \tan^2(x) + c$   
 D)  $4 \tan^4 x + c$   
 E)  $\frac{\sec^3 x \tan^4 x}{12} + c$

15)  $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx =$

- A)  $\frac{\cos^2 \sqrt{x}}{2x} + c$   
 B)  $2 \sin \sqrt{x} + c$   
 C)  $\frac{1}{2} \sin \sqrt{x} + c$   
 D)  $-\frac{1}{2} \sin \sqrt{x} + c$   
 E)  $-2 \sin \sqrt{x} + c$

16)  $\int \sin 2\theta \cos 2\theta d\theta =$

- A)  $\frac{1}{4} \sin^2 2\theta + c$   
 B)  $\frac{1}{2} \sin^2 2\theta + c$   
 C)  $-\frac{1}{4} \sin^2 2\theta + c$   
 D)  $-\frac{1}{2} \sin^2 2\theta + c$   
 E)  $\sin^2 2\theta + c$

17)  $\int \frac{d\theta}{\cos^2 2\theta} =$

- A)  $\frac{1}{2} \cot 2\theta + c$   
 B)  $\frac{1}{2} \tan 2\theta + c$   
 C)  $-\frac{2}{\cos 2\theta} + c$   
 D)  $2 \tan 2\theta + c$   
 E)  $-\frac{1}{2} \cot 2\theta + c$

18)  $\int \sec^3 x \tan x dx =$

- A)  $\frac{\tan^2 x}{2} + c$   
 B)  $\frac{\sec^2 x}{2} + c$   
 C)  $\frac{\sec^4 x \tan^2 x}{8} + c$   
 D)  $\frac{\sec^4 x}{4} + c$   
 E)  $\frac{\sec^3 x}{3} + c$

19)  $\int \frac{\sin \frac{3}{\theta}}{\theta^2} d\theta =$

- A)  $\frac{3 \cos^2 \left(\frac{3}{\theta}\right)}{\theta^3} + c$   
 B)  $-\frac{1}{3} \cos \frac{3}{\theta} + c$   
 C)  $\frac{\sin^2 \left(\frac{3}{\theta}\right)}{6\theta^3} + c$   
 D)  $-3 \cos \frac{3}{\theta} + c$   
 E)  $\frac{1}{3} \cos \frac{3}{\theta} + c$

20)  $\int \cos(\cos x) \sin x dx =$

- A)  $-\sin(\sin x) + c$   
 B)  $-\sin(\cos x) + c$   
 C)  $\cos(\cos x) + c$   
 D)  $\sin(\cos x) + c$   
 E)  $-\sin x + c$

21)  $\int \frac{\cos 2\theta}{\sin^2 2\theta} d\theta =$

- A)  $-\frac{1}{3 \sin^3 2\theta} + c$   
 B)  $-\frac{1}{6 \sin^3 2\theta} + c$   
 C)  $2 \sin 2\theta + c$   
 D)  $\frac{1}{2 \sin 2\theta} + c$   
 E)  $-\frac{1}{2 \sin 2\theta} + c$