Unit #6: Parametric and Polar Derivatives
Topic: Parametric Derivatives
Objective: SWBAT solve free response questions using the derivatives of parametric equations.

Warm Up #2:

1989 BC #4 (**NO CALCULATOR**)

Consider the curve given by the parametric equations $x = 2t^3 - 3t^2$ and $y = t^3 - 12t$.

a) Find $\frac{dy}{dx}$, in terms of t.

b) Write an equation for the line tangent to the curve at the point where $t = -1$.

c) Find the x- and y-coordinates for each critical point on the curve and identify each point as having a vertical or horizontal tangent.

Problem Set #2: Read each question carefully and show all work.

1. *1984 BC #2* **NO CALCULATOR**

The path of a particle is given for time $t > 0$ by the parametric equations $x = t + \frac{2}{t}$ and $y = 3t^2$.

a) Find the coordinates of each point on the path where the velocity of the particle in the x direction is zero.

b) Find $\frac{dy}{dx}$ when $t = 1$.

c) Find $\frac{d^2y}{dx^2}$ when $y = 12$.

2. *2003 BC #4 Form B* **(NO CALCULATOR)**

A particle moves in the xy-plane so that the position of the particle at any time t is given by

$$x(t) = 2e^{3t} + e^{-7t} \qquad and \qquad y(t) = 3e^{3t} - e^{-2t}$$

a) Find $\frac{dy}{dx}$ in terms of t, and find $\lim_{t\to\infty} \frac{dy}{dx}$.

b) Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.

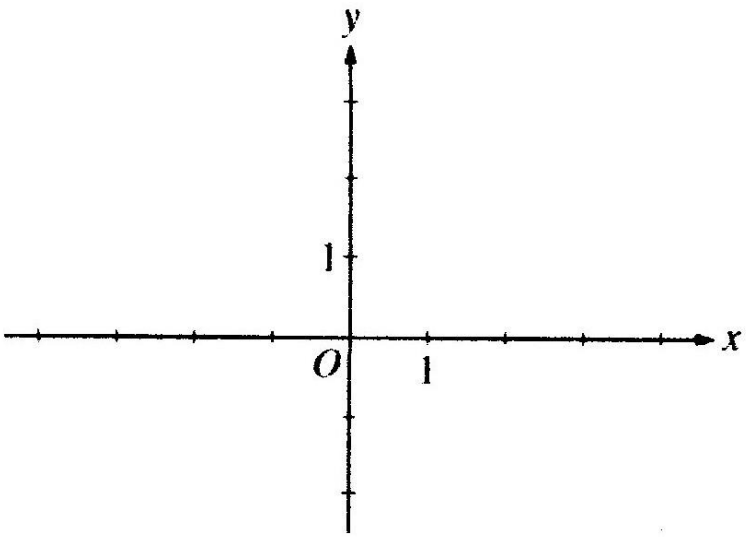
c) Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.

3. **CALCULATOR**

A particle moves in the xy-plane so that its position at any time , $0 \le t \le \pi$, is given by

$$x(t) = \frac{t^2}{2} - ln(1+t) \quad and \quad y(t) = 3sint$$

a) Sketch the path of the particle in the xy −plane below. Indicate the direction of motion along the path.



b) At what time t, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the coordinate of the point at this time?

c) At what time t, $0 < t < \pi$, is the particle on the $y - axis$?

4. *1974 BC #5* **(NO CALCULATOR)**

Given the parametric equations $x = 2(\theta - sin\theta)$ and $y = 2(1 - cos\theta)$:

a) Find $\frac{dy}{dx}$ in terms of θ.

b) Find an equation of the line tangent to the graph $\theta = \pi$.

c) Find an equation of the line tangent to the graph $\theta = 2\pi$.

d) Find $\frac{d^2y}{dx^2}$ in terms of θ.

Answers

WARM UP

a) $\frac{t^2-4}{2t^2-2t}$

b) $y-11=-\frac{3}{4}(x+5)$

c) (-28,16) horizontal, (0,0) vertical, (-1,-11) vertical, (4,-16) horizontal

1. a) $\left(2\sqrt{2}\,,6\right)$

   b) -6

   c) -24

2. a) $\frac{9e^{3t}+2e^{-2t}}{6e^{3t}-7e^{-7t}}$; limit is $\frac{3}{2}$ b) none c) $t=\frac{1}{10}ln\left(\frac{7}{6}\right)$

3. a) graph b) $t=0.618$; $(-0.290,1.738)$ c) $t=1.286$

4. a) $\frac{sin\theta}{1-cos\theta}$ b) $y=4$ c) $x=4\pi$ d) $-\frac{1}{(1-cos\theta)^2}$