Unit #3: Differential Equations
Topic: Separable Differential Equations
Objective: SWBAT find a particular solution to a separable differential equation.

Warm Up #2:

Find the particular solution to the equation $\frac{dy}{dx} = e^x - 6x^2$ whose graph passes through the point (1,0).

We cannot find a unique or particular solution to a differential equation unless we are given further information. If the general solution to a first-order differential equation is continuous, then all that is needed to write a particular solution is a single point. This given point is called the *initial condition*.

Example: 1985 BC #4

Given the differential equation $\frac{dy}{dx} = \frac{-xy}{lny}, y > 0$.

(a) Find the general solution of the differential equation.

(b) Find the particular solution that satisfies the condition that $y = e^2$ when $x = 0$. Express your answer in the form $y = f(x)$.

(c) Explain why $x = 2$ is not in the domain of the solution found in part (b).

Problem Set #2:

1) *2000 AB 6*

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

2) *2005 AB #6 (PART)*

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

3) *2002 (Form B) BC #5*

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at the point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

4) Find the particular solution if $\frac{dy}{dx} = ycosx$ and $y = 3$ when $x = 0$.

5) *1998 AB #4 (Part)*

Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the intial condition $f(1) = 4$.

6) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ with the intial condition $f(0) = 2$.

7) Find the particular solution $y = f(t)$ to the differential equation $\frac{dy}{dt} = \frac{1}{y^2}$ with the intial condition $y(2) = 3$.

8) Find $f(t)$ by solving the separable differential equation $\frac{dy}{dt} = ty$ with the intial condition $y(2) = 1$.

Answer Key:

1) a) $y = \frac{1}{2} ln(2x^3 + e)$ b) domain: $x > \left(-\frac{e}{2}\right)^{1/3}$ range: $(-\infty, \infty)$

2) $y = -\sqrt{-2x^2 + 3}$ 3) a) (3,-2) local max b) $y = -\sqrt{6x - x^2 + 16}$

4) $y = 3e^{sinx}$ 5) $y = \sqrt{x^3 + x + 14}$ 6) $y = \sqrt{e^{-x^2} + 3}$

7) $y = (3t + 21)^{1/3}$ 8) $y = e^{\frac{1}{2}t^2 - 2}$