Unit #7: Differentiation
Topic: The Product and Quotient Rules
Objective: SWBAT find the formula for the derivative of a function by using the product or quotient rule..

Warm Up #2:

Given the function $f(x) = x^2 - 3x$

a) Find the coordinates where the function has a tangent line with a slope of $-\frac{1}{2}$.

b) Find the coordinates where the function has a horizontal tangent line.

While the sum/difference of two functions is the sum/difference of their derivatives, the derivatives of the product/quotient of two functions **IS NOT** the product/quotient of their derivatives.

Rule 4 The Product Rule

Rule 5 The Quotient Rule

Examples:

1) Find $f'(x)$ if $f(x) = (x^2 + 1)(x^3 + 3)$.

2) Find $\frac{d}{dx}\left[\frac{x^2 - 1}{x^2 + 1}\right]$.

3) Suppose u and v are both functions that are differentiable at $x = 2$. If $u(2) = 3$, $u'(2) = -4$, $v(2) = 1$, *and* $v'(2) = 5$, find each of the following derivatives at $x = 2$,

a) $\frac{d}{dx}[uv]$

b) $\frac{d}{dx}\left[\frac{u}{v}\right]$

$$(ab)' = a'b + ab' \qquad \left(\frac{a}{b}\right)' = \frac{a'b - ab'}{b^2}$$

In Exercises 1–6, find dy/dx.

1. $y = -x^2 + 3$
2. $y = \frac{x^3}{3} - x$
3. $y = 2x + 1$
4. $y = x^2 + x + 1$
5. $y = \frac{x^3}{3} + \frac{x^2}{2} + x$
6. $y = 1 - x + x^2 - x^3$

In Exercises 7–12, find the horizontal tangents of the curve.

7. $y = x^3 - 2x^2 + x + 1$
8. $y = x^3 - 4x^2 + x + 2$
9. $y = x^4 - 4x^2 + 1$
10. $y = 4x^3 - 6x^2 - 1$
11. $y = 5x^3 - 3x^5$
12. $y = x^4 - 7x^3 + 2x^2 + 15$
13. Let $y = (x + 1)(x^2 + 1)$. Find dy/dx **(a)** by applying the Product Rule, and **(b)** by multiplying the factors first and then differentiating.
14. Let $y = (x^2 + 3)/x$. Find dy/dx **(a)** by using the Quotient Rule, and **(b)** by first dividing the terms in the numerator by the denominator and then differentiating.

In Exercises 15–22, find dy/dx

15. $(x^3 + x + 1)(x^4 + x^2 + 1)$
16. $(x^2 + 1)(x^3 + 1)$
17. $y = \frac{2x + 5}{3x - 2}$
18. $y = \frac{x^2 + 5x - 1}{x^2}$
19. $y = \frac{(x - 1)(x^2 + x + 1)}{x^3}$
20. $y = (1 - x)(1 + x^2)^{-1}$
21. $y = \frac{x^2}{1 - x^3}$
22. $y = \frac{(x + 1)(x + 2)}{(x - 1)(x - 2)}$
23. Suppose u and v are functions of x that are differentiable at $x = 0$, and that $u(0) = 5$, $u'(0) = -3$, $v(0) = -1$, $v'(0) = 2$. Find the values of the following derivatives at $x = 0$.

(a) $\frac{d}{dx}(uv)$ (b) $\frac{d}{dx}\left(\frac{u}{v}\right)$

(c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ (d) $\frac{d}{dx}(7v - 2u)$

24. Suppose u and v are functions of x that are differentiable at $x = 2$ and that $u(2) = 3$, $u'(2) = -4$, $v(2) = 1$, and $v'(2) = 2$. Find the values of the following derivatives at $x = 2$.

(a) $\frac{d}{dx}(uv)$ (b) $\frac{d}{dx}\left(\frac{u}{v}\right)$

(c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ (d) $\frac{d}{dx}(3u - 2v + 2uv)$

25. Which of the following numbers is the slope of the line tangent to the curve $y = x^2 + 5x$ at $x = 3$?

i. 24 **ii.** $-5/2$ **iii.** 11 **iv.** 8

26. Which of the following numbers is the slope of the line $3x - 2y + 12 = 0$?

i. 6 **ii.** 3 **iii.** 3/2 **iv.** 2/3

In Exercises 27 and 28, find an equation for the line tangent to the curve at the given point.

27. $y = \frac{x^3 + 1}{2x}, x = 1$
28. $y = \frac{x^4 + 2}{x^2}, x = -1$

In Exercises 29–32, find dy/dx.

29. $y = 4x^{-2} - 8x + 1$
30. $y = \frac{x^{-4}}{4} - \frac{x^{-3}}{3} + \frac{x^{-2}}{2} - x^{-1} + 3$
31. $y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$
32. $y = 2\sqrt{x} - \frac{1}{\sqrt{x}}$

In Exercises 33–36, find the first four derivatives of the function.

33. $y = x^4 + x^3 - 2x^2 + x - 5$
34. $y = x^2 + x + 3$
35. $y = x^{-1} + x^2$
36. $y = \frac{x + 1}{x}$

In Exercises 37–42, support your answer graphically.

37. Find an equation of the line perpendicular to the tangent to the curve $y = x^3 - 3x + 1$ at the point (2, 3).
38. Find the tangents to the curve $y = x^3 + x$ at the points where the slope is 4. What is the smallest slope of the curve? At what value of x does the curve have this slope?
39. Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ where the tangent is parallel to the x-axis.
40. Find the x- and y-intercepts of the line that is tangent to the curve $y = x^3$ at the point $(-2, -8)$.

In Exercises 41-42 , find dy/dx.

41. $y = (3x - 2x^2)(4 + 5x)$
42. $y = (x^4 + x - 5)(3x^2 - 6)$