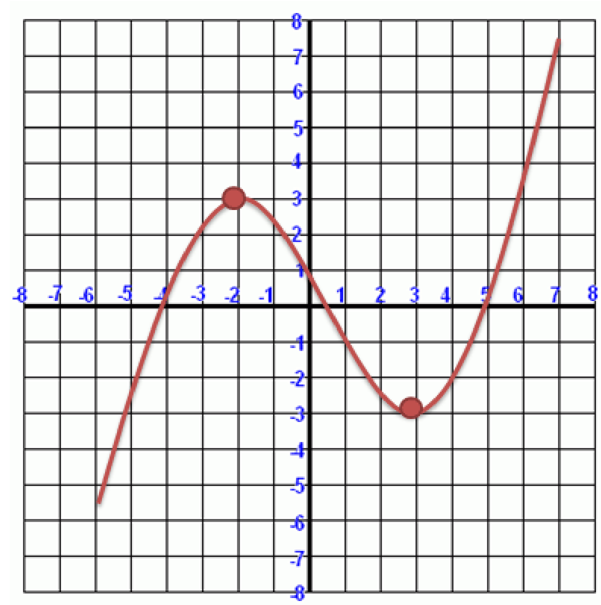
Unit #10: Applications of Differentiation
Topic: Extreme Values
Objective: SWBAT identify the extrema of a function on an interval.

Warm Up #2:



A graph of $f(x)$ is given at the right.

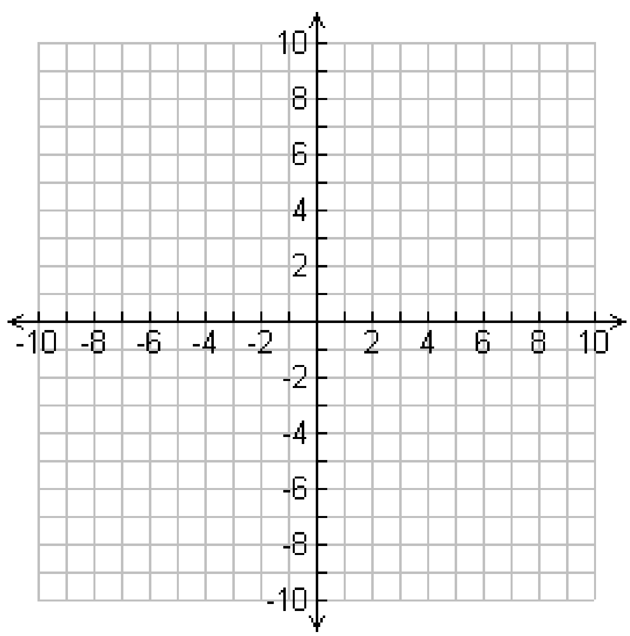
1) On what interval(s) is $f(x)$ increasing? decreasing?

2) Does $f(x)$ have any relative minimum or maximum points? If so, what are they?

3) What do you notice about the derivative at the relative extrema points? Describe the behavior of the derivative around these points.

4) On what interval(s) is $f(x)$ concave up/down?

5) Sketch the graph of $f'(x)$.



6) What do you notice about the second derivative at the point of inflection of $f(x)$? Describe the behavior of the second derivative around this point.

Absolute/Global Extrema:

If f is a function on an interval I, then

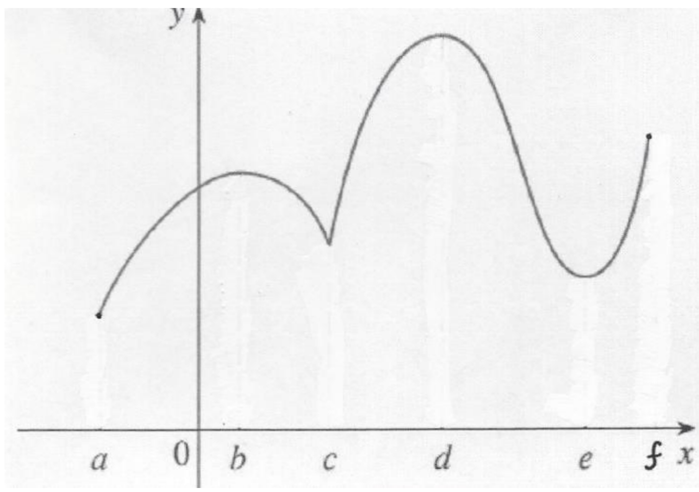
f has an ______________________________ at c if and only if

______________________________.

f has an ______________________________ at c if and only if

______________________________.

Example #1: Does the following graph have an absolute minimum/maximum value? If so, where?



For each of the following, use a **Graphing Calculator** to find all points of absolute minima/maxima on the given interval.

a) $y = x^2 + 1 \, ; [-1,2]$	b) $y = \frac{8}{x^2+4}$; $[0, 5]$
c) $y = x^3 - 2$; $x \geq -2$	d) $y = (5x + 25)^{1/3}$; $(-\infty, 4]$

The Extreme Value Theorem (EVT)

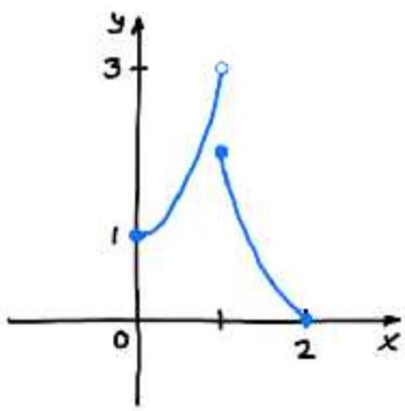
If f is continuous on a ________________ interval $[a, b]$, then f has BOTH a

_________________________ and _________________________ value on the interval.

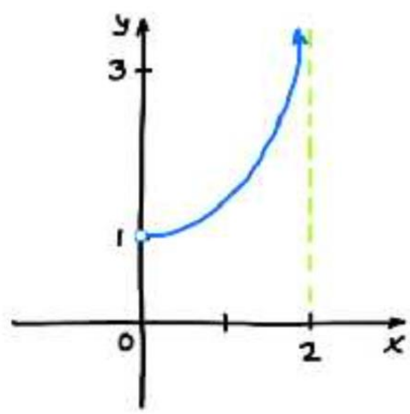
Example #2:

Determine if the EVT applies for each of the function on the interval $[0,2]$. If so, find the extrema. If not, explicitly state why, then determine if the function happens to still have extrema on the interval.

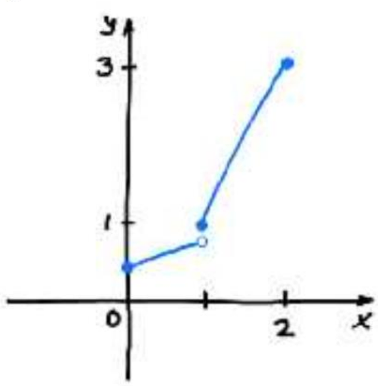
(a)



(b)



(c)



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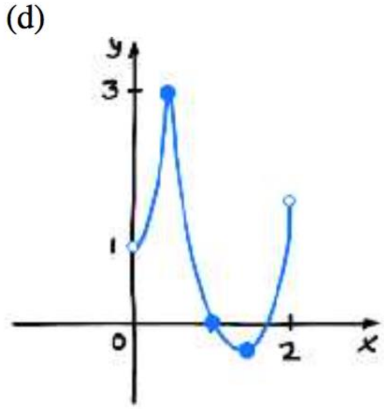


Critical Values

A ___________________________ of a function f is a value $x = c$ in the domain of f such that either

___________________________ OR ___________________________

Before we find extreme values analytically by analyzing the equation or graph of a function, we need to consider the following:

Theorem

Absolute/Global extrema can only occur at a ______________________________

or at an ______________________________ *of an interval.*

Relative/Local Extrema

A function f has a local(relative) ____________________ at c if ____________________

when x is ________________ c.

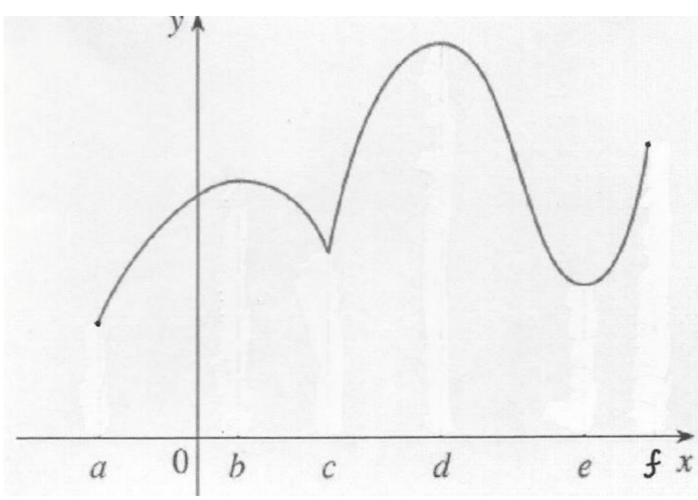
A function f has a local(relative) ____________________ at c if ____________________

when x is ________________ c.

Local/Relative extrema can ***only*** *occur at a* ____________________________!!

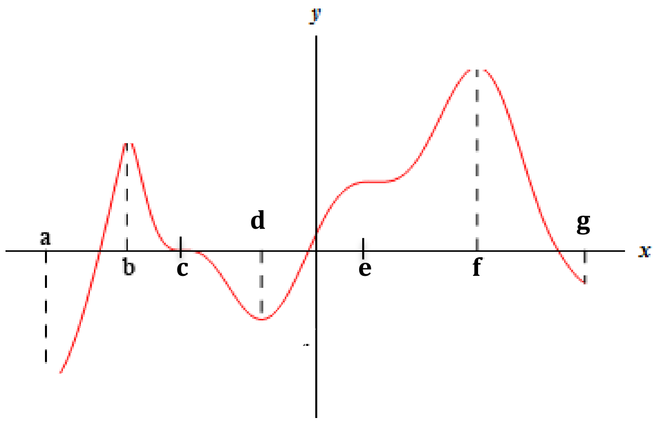
Local/Relative extrema ____________________ *occur at an* ____________________ *of an interval!!*

Example #3: Identify all the critical values of the graph below, then determine whether a local max, local min, or neither occurs at that critical value.



Problem Set #2: For each of the following, identify any absolute minima/maxima and all critical values and determine whether there is a local min/max or neither at that critical value. A **Calculator** may be used.

1)



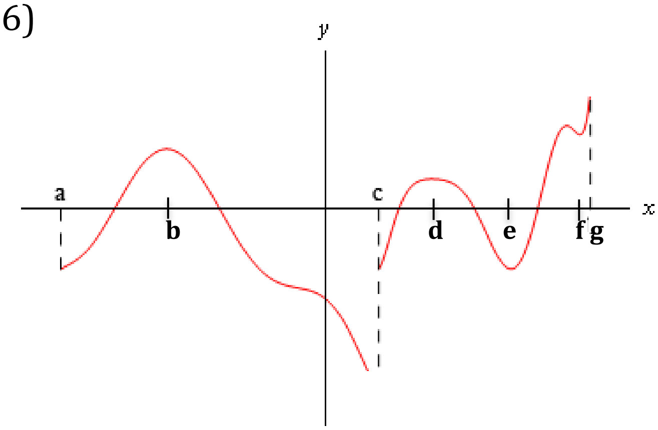
2) $y = 2 - |x - 4|\ ;\ x \geq 1$

3) $y = 2x - 3x^{\frac{2}{3}}\ ;\ x \leq 4$

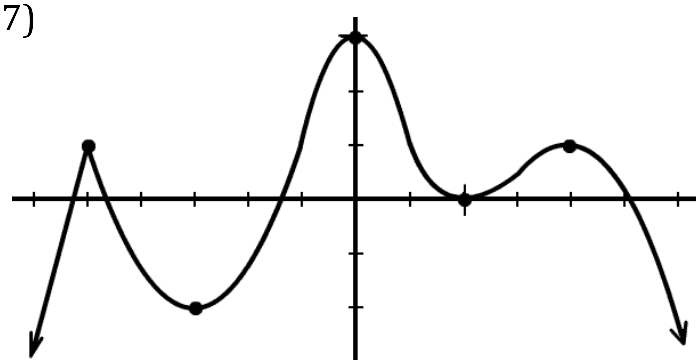
4) $g(x) = -\frac{1}{6}(x+1)^{\frac{7}{3}} + \frac{14}{3}(x+1)^{\frac{1}{3}}\ ;\ (-5,0)$

5) $y = \frac{1}{2}x^4 - x^3 - x^2 + 2\ ;\ [-2,4]$

6)



7)



8) $f(x) = \begin{cases} 5 - 2x^2, & x < 1 \\ x - 1, & x \geq 1 \end{cases}$

9)