Unit #7: Sequence and Series Date: ________________

Topic: Tests for Convergence

Objective: SWBAT determine whether a series converges or diverges.

Warm Up #3:

1) Find the first four terms of the sequence $a_n = \frac{(-2)^{n+1}}{(n+2)}$.

2) If $s_n = \left(\frac{(5+n)^{100}}{5^{n+1}}\right)\left(\frac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

Before we look at specific tests for convergence we need to remember the following:

Series known to converge:

1. A geometric series with $|r| < 1$ converges
2. A repeating decimal series converges
3. A Telescoping series converges
4. A p-series with $p > 1$ converges

The following tests should be used in the order given.

1. The Divergence Test

If $\lim_{n\to\infty} a_n \neq 0$, **then** the series DIVERGES!

BUT

If $\lim_{n\to\infty} a_n = 0$, **then** the test is inconclusive.

Example #1:

a) $\sum_{n=1}^{\infty} \frac{2n}{n+2}$

b) $\sum_{n=1}^{\infty} \frac{1}{n}$ What series is this?

2. The Integral Test

If $f(x)$ is a CONTINUOUS, POSITIVE, and DECREASING function for $x \geq 1$ and $a_n = f(x)$,

Then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either BOTH converge or diverge.

NOTE: This does not mean that the series converges to the value of the definite integral!!

Example #2: Do each of the following series converge or diverge?

a) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

b) $\sum_{n=1}^{\infty} \frac{n}{e^n}$

Practice Problems: Determine whether each of the following series converges or diverges.

1) $\sum_{n=1}^{\infty} \frac{n-1}{2n+1}$	2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$
3) $\sum_{n=1}^{\infty} 9n^3 e^{-n^4}$	4) $\sum_{n=1}^{\infty} \frac{3n^2}{\sqrt{n^3+8}}$
5) $\sum_{n=1}^{\infty} n!\, e^{-n}$	6) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

7) $\sum_{n=0}^{\infty} \frac{n^2+2n}{e^n}$	8) $\sum_{n=1}^{\infty} \frac{1+5^n}{1+2^n}$
9) $\sum_{n=1}^{\infty} \frac{n^2}{e^{n^3}}$	10) $\sum_{n=1}^{\infty} \frac{1}{1+10n}$
11) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$	12) $\sum_{n=1}^{\infty} \frac{lnn}{n^2}$
13) $\sum_{n=1}^{\infty} \frac{3^n-2}{3^n}$	14) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

Warm Up #4:

Which of the following series diverge?

I. $\sum_{n=0}^{\infty}\left(\frac{sin2}{\pi}\right)^n$ II. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$ III. $\sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$

(A) III only (B) I and II only (C) I and III only
(D) II and III only (E) I, II, and III

3. The Ratio Test

Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero terms and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$

Then

- If $L < 1$ then the series ______________________________.
- If $L > 1$ then the series ______________________________.
- If $L = 1$ then the test is ______________________________.

Example #3: Do each of the following series converge or diverge?

a) $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$

b) $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

Practice Problems: Determine whether each of the following series converges or diverges.

15) $\sum_{n=1}^{\infty} \frac{2^n}{3^n+1}$	16) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$
17) $\sum_{n=0}^{\infty} \frac{n^{10}}{10^n}$	18) $\sum_{n=0}^{\infty} n^2 e^{-n}$

19) $\sum_{n=2}^{\infty} \frac{1}{nln(n)}$	20) $\sum_{n=1}^{\infty} \frac{3^n}{n^3 2^n}$
21) $\sum_{n=1}^{\infty} \frac{n^2-1}{2^n}$	22) $\sum_{n=0}^{\infty} \frac{n^3+2}{9^n}$
23) $\sum_{n=1}^{\infty} \frac{n+1}{n4^n}$	24) $\sum_{n=0}^{\infty} \frac{n!}{6^n}$

Answer Key

1) Diverges
2) Diverges
3) Converges
4) Diverges
5) Diverges
6) Converges
7) Converges
8) Diverges
9) Converges
10) Diverges
11) Diverges
12) Converges
13) Diverges
14) Converges

15) Converges
16) Ratio Test is Inconclusive/ Diverges by Integral Test
17) Converges
18) Converges
19) Ratio Test is Inconclusive/ Diverges by Integral Test
20) Diverges
21) Converges
22) Converges
23) Converges
24) Diverges