*Unit #3:* Differential Equations
*Topic:* Slope Fields
*Objective:* *SWBAT graph a general solution to a differential equation by using a slope field.*
*SWBAT match a slope field to the correct differential equation.*

# Warm Up #3:

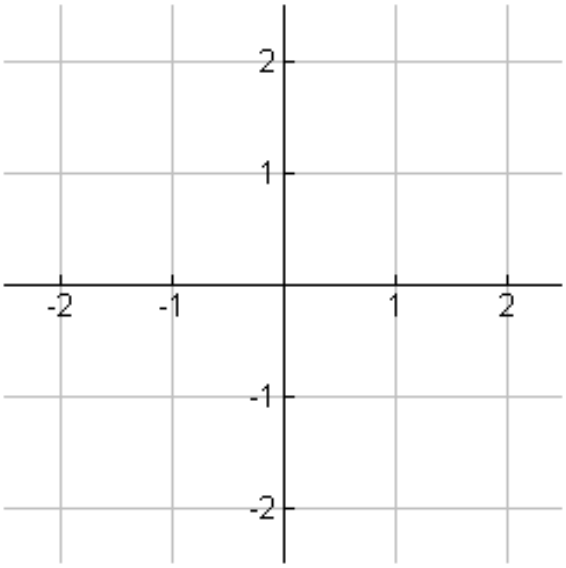
Find the particular solution to the equation $\frac{dy}{dx} = x^2$ whose graph passes through the point $(0, -1)$.

*Definition:*
A **Slope Field** is a graphical general solution to a differential equation.

*Example #1:*

a) Draw a slope field for the differential equation: $\frac{dy}{dx} = x^2$ at the indicated points.

b) Draw a particular solution at $(0, -1)$.



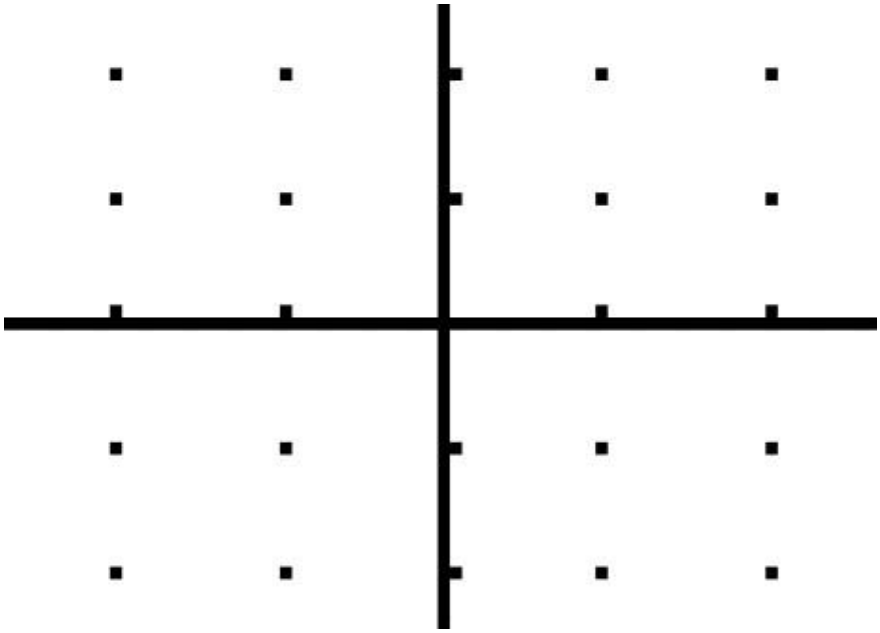

Many times, differential equations are **NOT** explicit functions of a single variable, and sometimes they are not even solvable by analytic methods. That's where slope fields come in.☺

*When drawing slope fields:*

1) Draw the piece of the tangent line at a point long enough to see, but not so long that it interferes with the other tangent lines.
2) Be sure your slopes of 0, 1, -1, and ∞ are spot on. All other slopes must be at a steepness relative to these slopes and the others around it.

*Example #2:* Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$.

a) On the axes provided, sketch a slope field for the given differential equation.

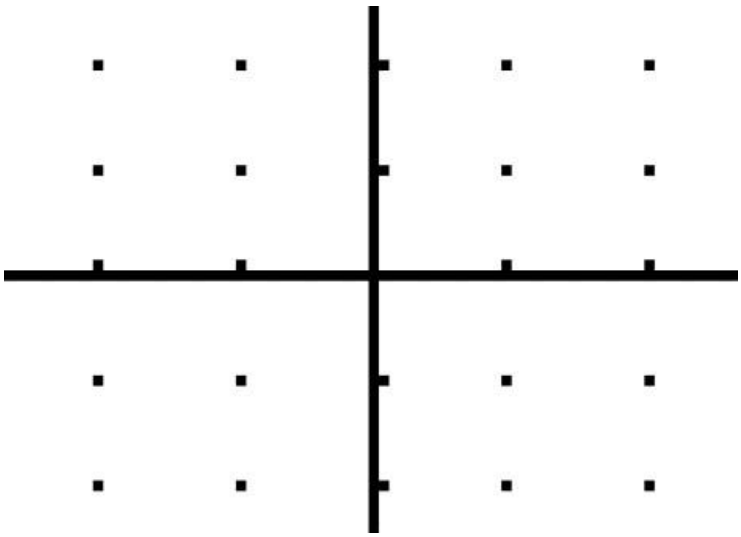


b) Sketch the solution curve that passes through the point (2,0).

*Problem Set #3:*

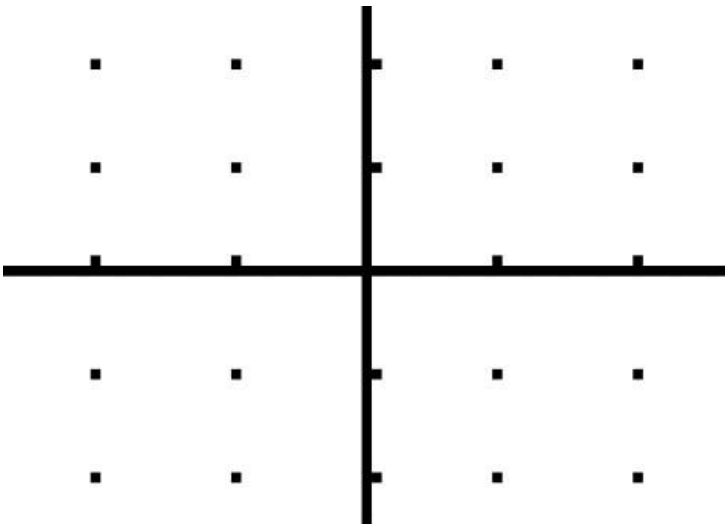
1) Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

a) On the axes provided, sketch a slope field for the given differential equation.



b) While only part of the slope field in part (a) is drawn, it is defined at every point in the $xy$-plane. Describe all points in the $xy$ −plane for which the slopes are negative.

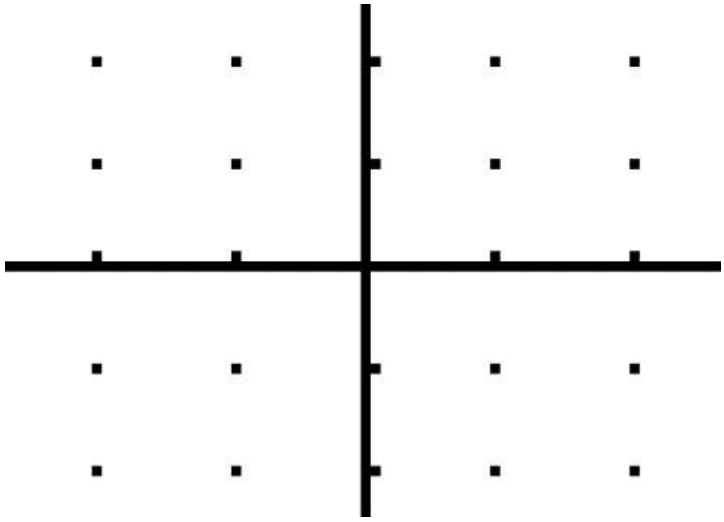
2) a) Draw the slope field for the differential equation $\frac{dy}{dx} = x^2y$.



b) Sketch the solution curve that passes through the point (0,-2).

3) Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

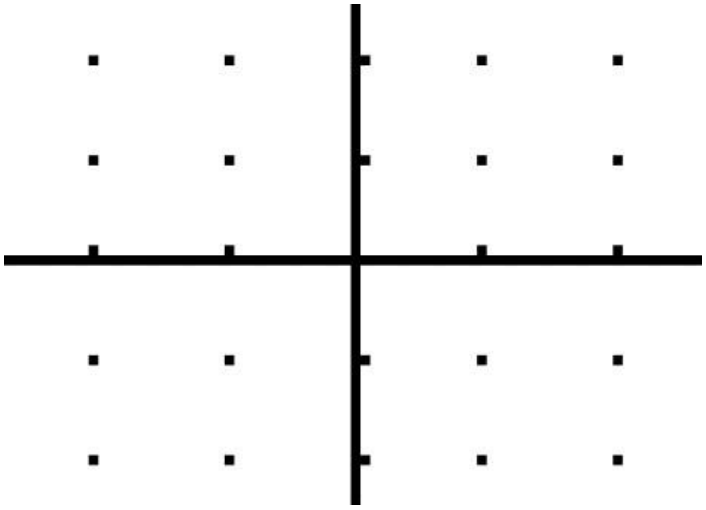
a) On the axes provided, sketch a slope field for the given differential equation.



b) Sketch the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = -1$.

4) Consider the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$.

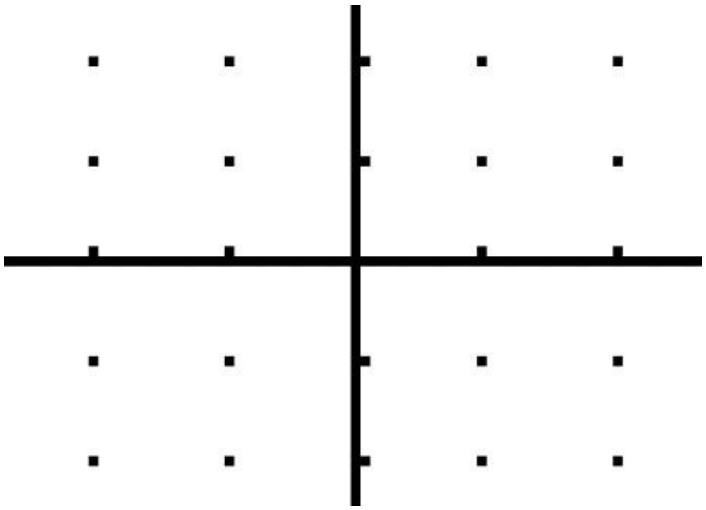
a) On the axes provided, sketch a slope field for the given differential equation.



b) Sketch the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = -2$?

5) Consider the differential equation $\frac{dy}{dx} = x + y$.

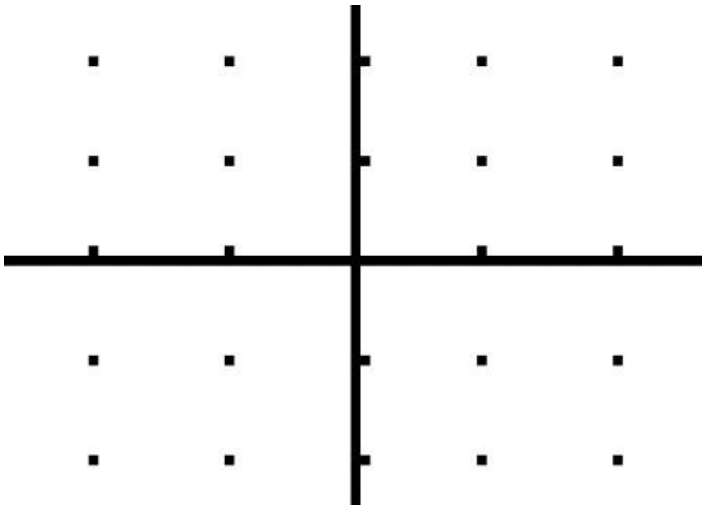
a) On the axes provided, sketch a slope field for the given differential equation.



b) Sketch the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$?

6) Consider the differential equation $\frac{dy}{dx} = y - 1$.

a) On the axes provided, sketch a slope field for the given differential equation.

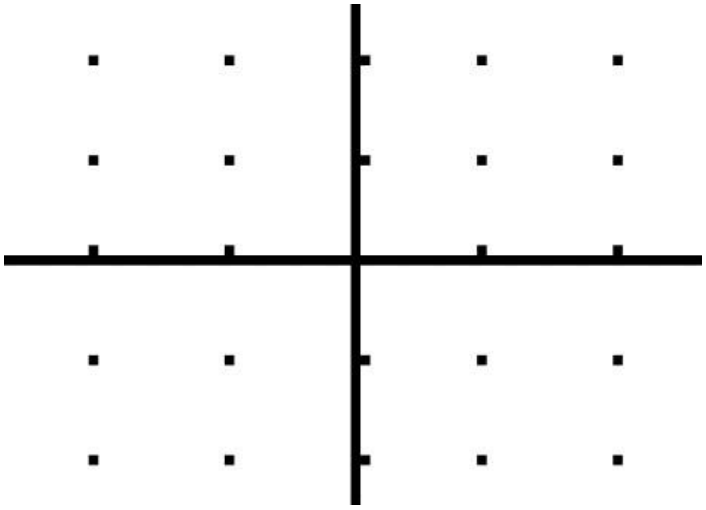


b) Sketch the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$?

# Warm Up #4:

Consider the differential equation $\frac{dy}{dx} = \frac{xy}{2}$.

a) On the axes provided, sketch a slope field for the given differential equation.



b) Let $f$ be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point $(1,1)$ and then use the equation to estimate the value of $f(1.2)$.

c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$ and then use your solution to find $f(1.2)$.

d) Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

*Example #3:* Which of the following differential equations matches the slope field given?

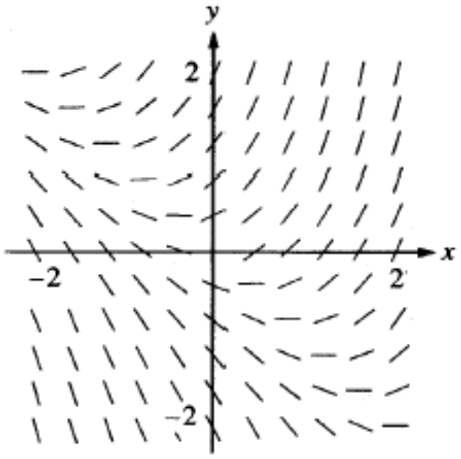
A) $\frac{dy}{dx} = 1 + x$

B) $\frac{dy}{dx} = x^2$

C) $\frac{dy}{dx} = x + y$

D) $\frac{dy}{dx} = lny$

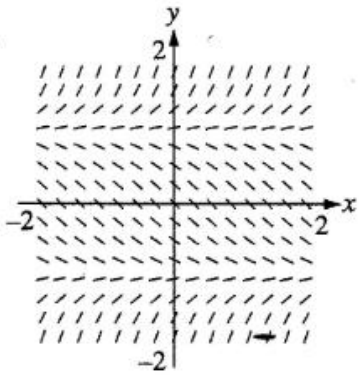
E) $\frac{dy}{dx} = \frac{x}{y}$



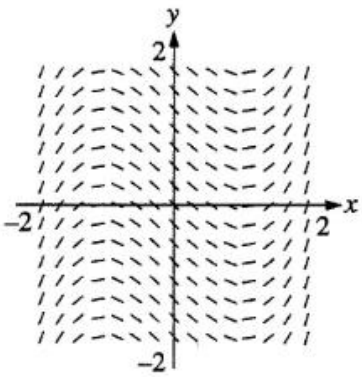

*Example #4:*

Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

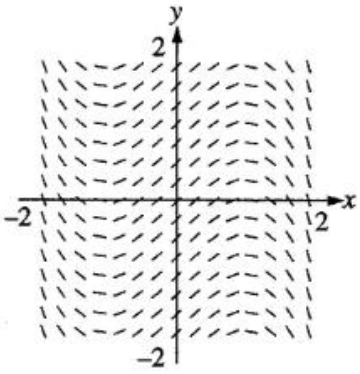
(A)



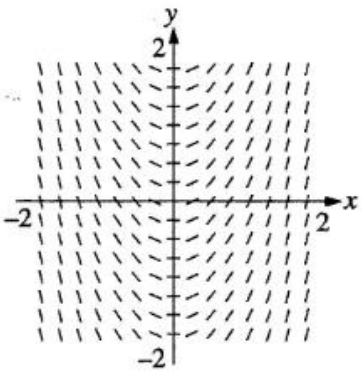

(B)



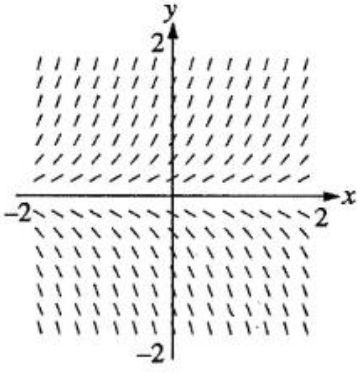

(C)




(D)



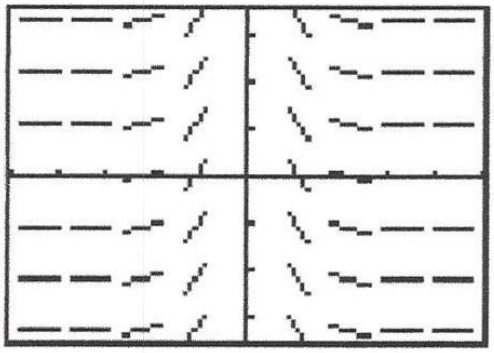

(E)



*Problem Set #4:*

For 7 – 12, match each slope field with the equation that the slope field could represent.

7.



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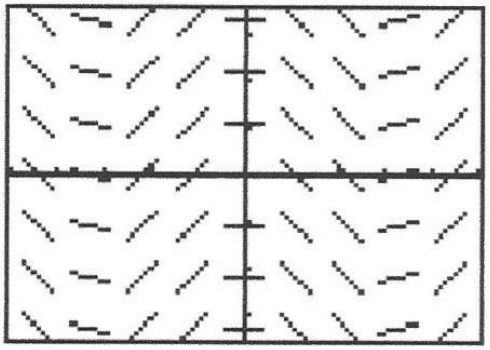

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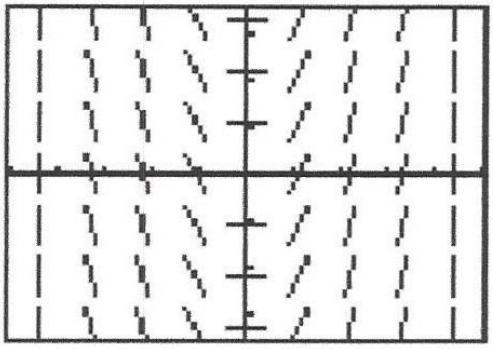

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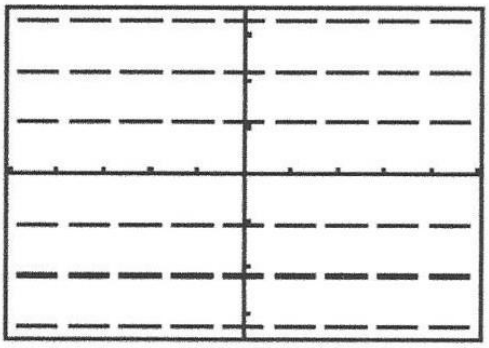

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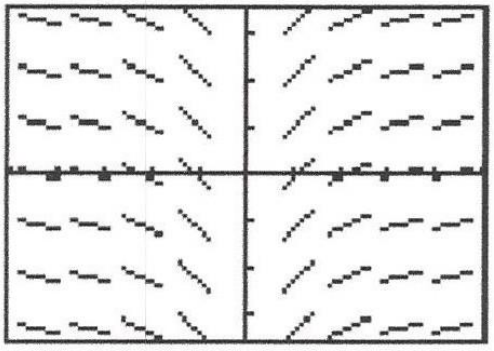

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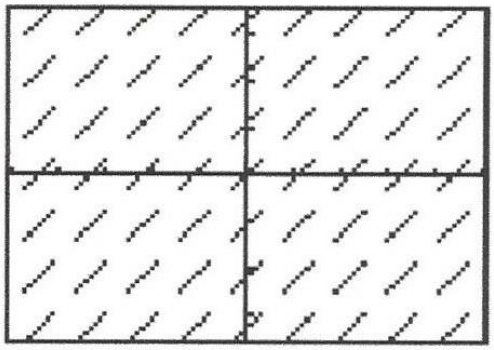


(A) $y = 1$

(B) $y = x$

(C) $y = x^2$

(D) $y = \frac{1}{6}x^3$

(E) $y = \frac{1}{x^2}$

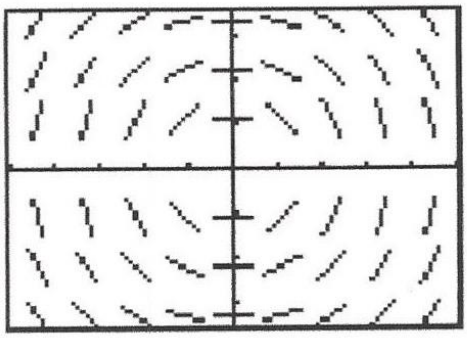
(F) $y = \sin x$

(G) $y = \cos x$

(H) $y = \ln|x|$

For 13 – 16, match the slope fields with their differential equations.

13.



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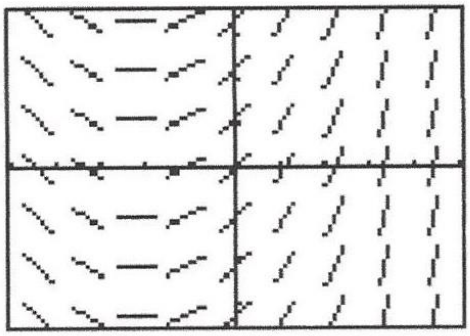

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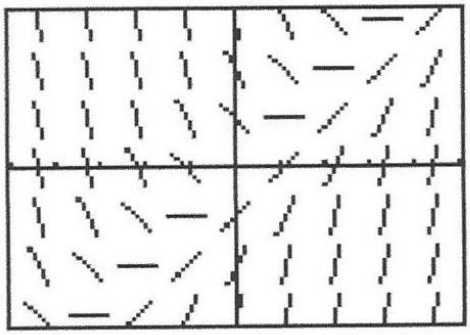

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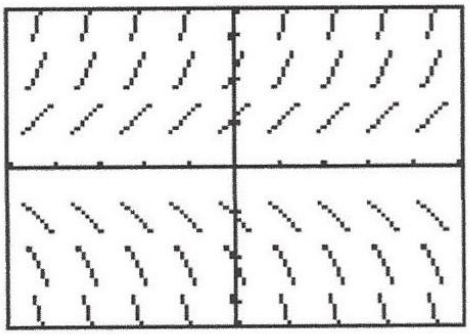

(A) $\frac{dy}{dx} = \frac{1}{2}x + 1$

(B) $\frac{dy}{dx} = x - y$

(C) $\frac{dy}{dx} = y$

(D) $\frac{dy}{dx} = -\frac{x}{y}$