Unit #6: Parametric and Polar Derivatives
Topic: Arc Length
Objective: SWBAT find the arc length of a parametric curve

Warm Up #3:

Write an integral that can be used to find the length of the curve $y = ln(sinx)$ from $x = 1$ to $x = a$, where $1 < a < \pi$.

Arc Length of a Parametric Curve

If a smooth curve $x = f(t),\ y = g(t),\ a \le t \le b$, is transversed exactly once as t increases from a *to* b, the curve's length is

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Example #1: Find the length of the curve described by the following parametric equations on the given interval.

$$x = 3sint \quad and \quad y = 3cost\,, \quad 0 \le t \le \pi$$

Example #2: Write an integral that can be used to find the length of the curve described by the following parametric equations on the given interval.

$$x = sin(t^3) \quad and \quad y = e^{5t} \ \text{ from } \ t = 0 \ to \ t = \pi$$

Problem Set #3A: Find the length of the curve for each of the following ***without*** *a calculator.*

1. $x = 8cost + 8tsint\ ,\ y = 8sint - 8tcost\ ;\ 0 \le t \le \frac{\pi}{2}$

2. $x = \frac{(2t+3)^{3/2}}{3}\ ,\ y = t + \frac{t^2}{2}\ ;\ 0 \le t \le 3$

3. $x = 3t - t^3\ ,\ y = 3t^2\ ;\ 0 \le t \le 2$

4. $x = e^t - t\ ,\ y = 4e^{t/2}\ ;\ -8 \le t \le 3$

*Problem Set #3B: Find the length of the curve for each of the following **with** a calculator.*

5. $x = 1 + e^t$, $y = t^2$; $-3 \le t \le 3$

6. $x = lnt$, $y = \sqrt{t+1}$; $1 \le t \le 5$

7. $x = t^3$, $y = t^2 + 1$; $-1 \le t \le 1$

8. $x = t^2 + 1$, $y = \sqrt{t+2}$; $-1 \le t \le 2$

9. $x = t^2 + 3$, $y = cost$; $from\ (3,1)\ to\ (7, cos2)$

Answers

1. π^2 2. 21/2 3. 14 4. $e^3 + 11 - e^{-8}$ 5. 30.528 6. 1.931 7. 2.879 8. 5.204 9. 4.254