

Unit #4: Area and Volume

Topic: Finding the Area Between Two Curves

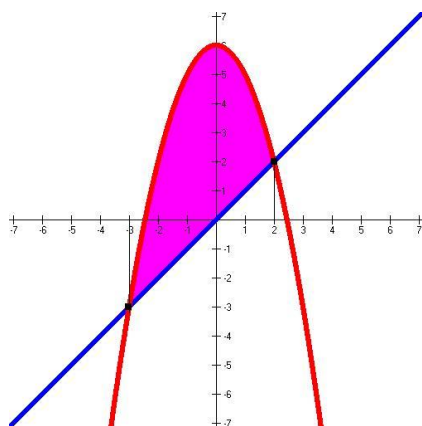
Objective: *SWBAT find the area between two curves by using integration.***Warm Up #3:**

What is the area of the region between the graph of $y = 4x^3 + 2$ and the x -axis from $x = 1$ to $x = 2$?

What if...?

We have seen previously that you can use integrals to find the area of the region between a curve and the x -axis, but what if we want to find the area between the two functions shown below?

How can we do this?

**Finding the Area between Two Functions**

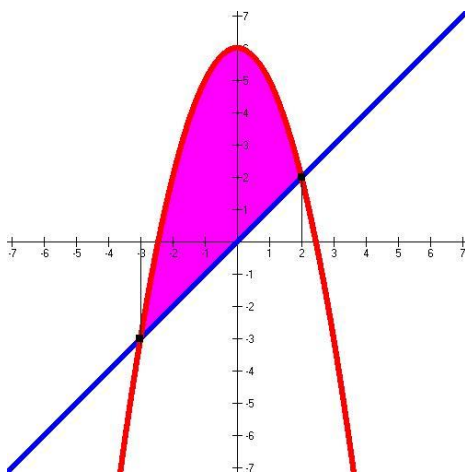
If f and g are continuous functions on the interval $[a,b]$, and if $f(x) \geq g(x)$ for all x in $[a,b]$, then the area of the region bounded above by $y = f(x)$, below by $y = g(x)$, on the left by $x = a$, and on the right by $x = b$ is:

$$A = \int_a^b \left[\underset{\substack{\uparrow \\ \text{Top}}}{f(x)} - \underset{\substack{\uparrow \\ \text{Bottom}}}{g(x)} \right] dx$$

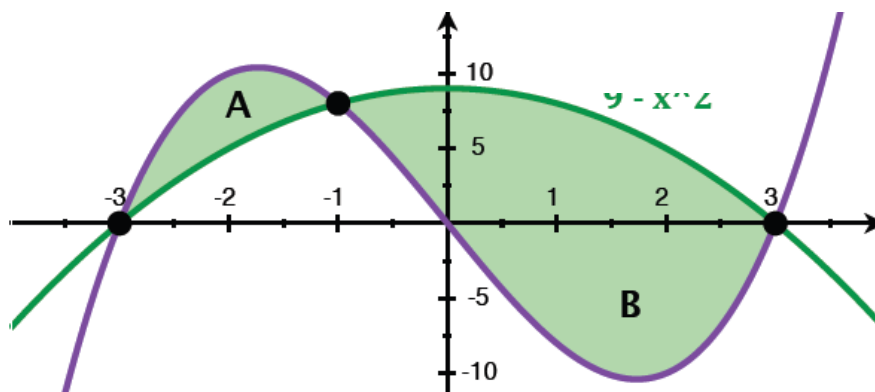


Model Problem #1:

Find the area of the region bounded by $f(x) = 6 - x^2$ and $g(x) = x$, as shown in the diagram below.

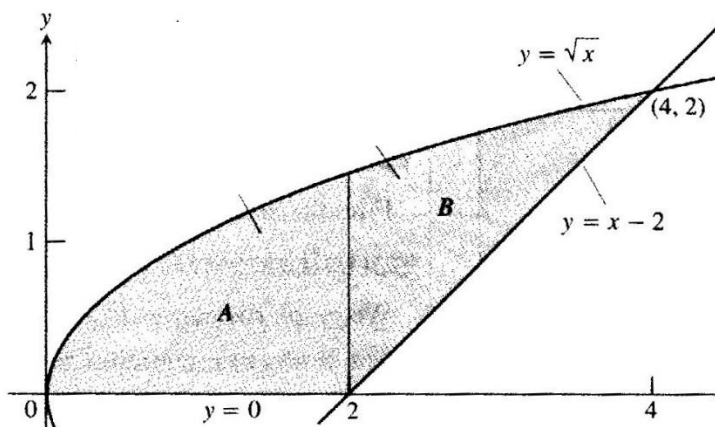
*Model Problem #2:*

Find the area bounded by the two curves $y = x^3 - 9x$ and $y = 9 - x^2$.



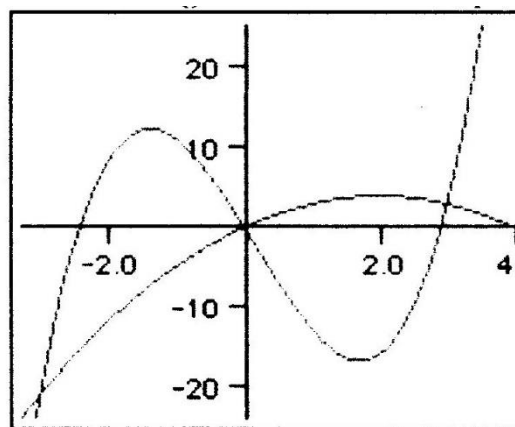
**Model Problem #3:**

Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$.

**Problem Set #3:**

1. Find the area of the region bounded by $y = x^2 - 2x$ and $y = 4 - x^2$.

2. Find the area of the region bounded by $y = 2x^3 - x^2 - 14x$ and $y = 4x - x^2$.

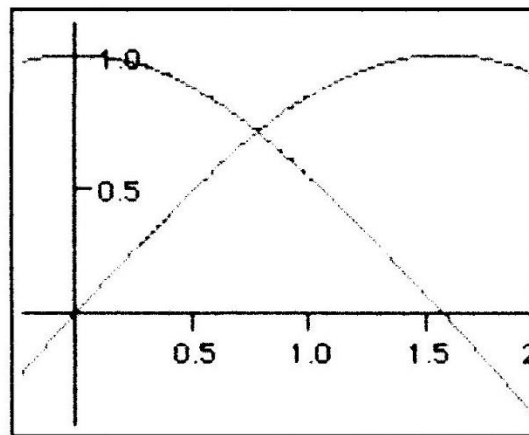


3. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?
(*Calculator required*)

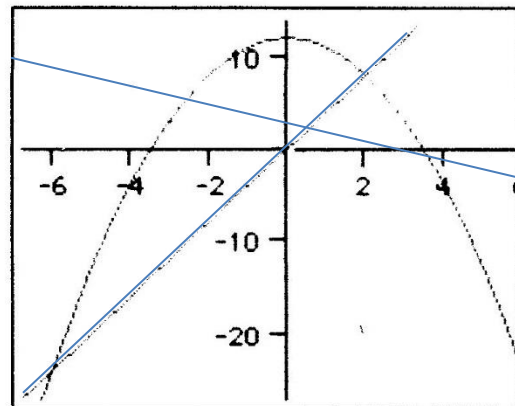
4. The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) 1

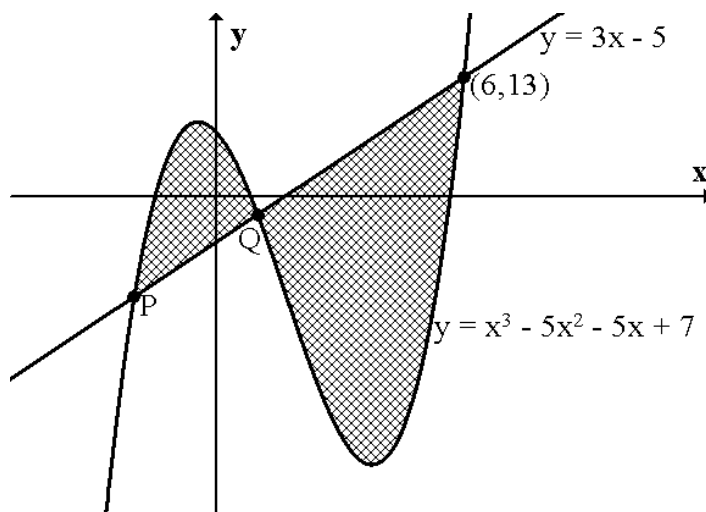
5. Find the region bounded by the graphs $x = 0$, and the first intersection of $y = \sin x$ and $y = \cos x$.



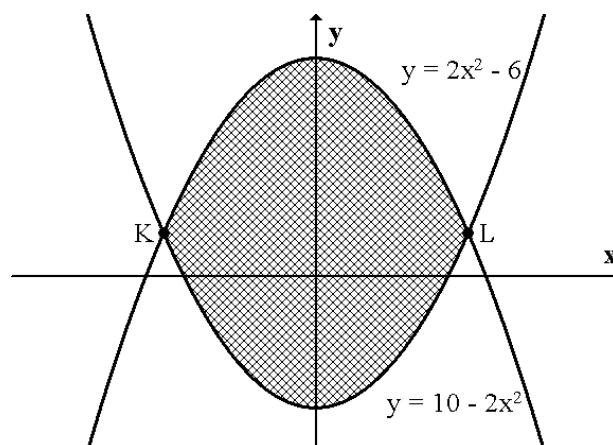
6. Find the area of the region bounded by the graphs $y = 12 - x^2$, $y = -x$, and $y = 4x$.



7. What is the area enclosed by the curves $y = x^3 - 5x^2 - 5x + 7$ and $y = 3x - 5$?



8. Find the area of the region bounded by the graphs of $y = 2x^2 - 6$ and $y = 10 - 2x^2$.



9. What is the area enclosed by the curves $y = \sin x$ and $y = 1 - x^2$?
(*Calculator required*)

Answer Key:

- 1) 9 square units
- 2) 81 square units
- 3) $71/6$ square units
- 4) A
- 5) $\sqrt{2} - 1$ square units
- 6) $320/3$ units squared
- 7) $863/3$ units squared
- 8) $128/3$ square units
- 9) 1.670 units squared



