

Unit #6: Continuity

Topic: Rates of Change

Objective: *SWBAT explain average rate of change.*

*SWBAT find the equation of a tangent and normal line to a curve at an indicated point.*

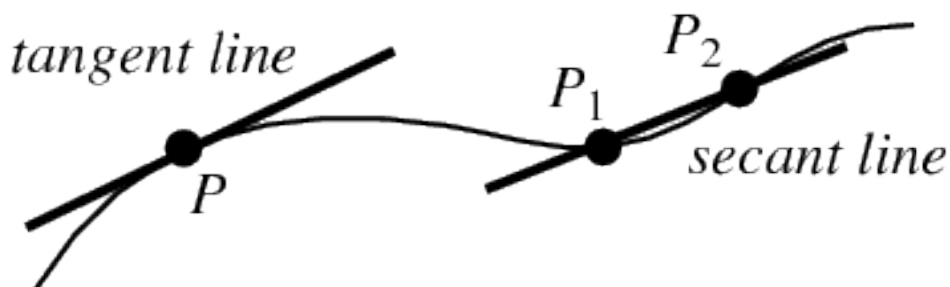
### Warm Up #3:

Write the equation of the line containing the points (1,3) and (-2,4).

The slope of a line is the constant rate of change of the linear function and can be found using any two points on the line,

*BUT...*

If we consider a function whose graph is a curve where the slope is not a constant we need to use secant and tangent lines to the curve to help us describe what is happening with the rate of change.



### Average Rate of Change

The *average rate of change* of a function on an interval  $[a, b]$  is the slope of the secant line through the two endpoints of the interval.

$$\text{Avg. Rate of Change} = \frac{f(b) - f(a)}{b - a}$$

*Example #1:*

Find the average rate of change of  $f(x) = x^3 - x$  over the interval  $[1,3]$ .

## The Equation of a Tangent Line to a Curve

The tangent line to a curve at a given point gives us the linear function that best fits the graph at the point of tangency and whose slope is the *instantaneous rate of change* of the curve.

The Slope of a Curve at a Point is the number

$$m = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

*Example #2:*

Write an equation for the equation of the tangent line to the curve  $f(x) = x^2 + 1$  at  $x = -2$ .

The *normal line* to a curve at a point is the line perpendicular to the tangent line at that point.

*How is the slope of the tangent line related to the slope of the normal line?*

*Example #3:*

Write an equation for the normal line to the curve  $f(x) = 4 - x^2$  at  $x = 1$ .

*Problem Set #3:*

1) Find the average rate of change of the function over each interval.

a)  $f(x) = x^3 + 1$ ;  $[2,3]$

b)  $f(x) = \sqrt{4x + 1}$ ;  $[0,2]$

c)  $f(x) = \cos x$ ;  $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

d)  $f(x) = e^x$ ;  $[1,3]$

e)  $f(x) = x^2 - 3x - 1$ ;  $[1,5]$

f)  $f(x) = \frac{1}{x-1}$ ;  $[2,5]$

2) For each of the following functions, write the equation of the tangent and normal line to the curve at the indicated point.

a)  $g(x) = x^2 + 5$  at  $x = -1$

b)  $f(x) = 2x - x^2$  at  $x = 3$

c)  $f(x) = x^2 + 3x - 10$  at  $x = 1$

d)  $f(x) = \sqrt{2+x} - 3$  at  $x = -1$

e)  $f(x) = (x+1)^2 + 1$  at  $x = 1$

