

Unit #1: Integration Review

Topic: Initial Value Problems

Objective: SWBAT find a particular solution to an indefinite integral by using an initial value.

Warm Up #3:

The volume V of a balloon is changing with respect to time t at a rate given by $\frac{dV}{dt} = 3t^{1/2} + \frac{1}{4}t$ ft³/sec. If, at $t = 4$, the volume is 20ft^3 , what does V equal?

An *initial-value problem* is a differential equation together with enough additional conditions to specify the constants of integration that appear in the general solution in order to obtain a *particular solution*.

Let's try another one:

The acceleration of a particle at time $t > 0$ moving along the x – axis is $a(t) = 3t + 2$ ft/sec². If at $t = 1$ seconds the velocity, $v(t) = 4$ ft/sec and the position, $x(t) = 6$ ft, then what is $x(2)$?

Problem Set #3: Find the particular solution for each of the following differential equations.

1) If $f'(x) = 12x^2 - 6x + 1$, $f(1) = 5$, then find $f(0)$.

2) If $\frac{dx}{dt} = \frac{4}{t} - \frac{1}{t^4} + 8$ and $x = 4$ when $t = 1$. Find $x(t)$.

3) If $\frac{d^2y}{dx^2} = 24x^2 - 10$ when $x = 1$, $\frac{dy}{dx} = 3$, and $y = 5$. Find the specific solution for y .

4) If $f'(x) = 6e^x - \cos x$, when $y = 4$ and $x = 0$, then find the particular solution for $f(x)$.

5) If $\frac{dy}{dx} = 9\sec^2 x - \frac{1}{5\sqrt{x}}$, when $y = 5$ and $x = 0$, then find y .

6) If $f'(x) = x(15x - 6)$, when $y = 4$ and $x = 1$, then find $y(2)$.

7) If $\frac{dv}{dt} = 6\sec t \tan t + 6t - 3e^t$ and $v(0) = 5$, then find $v(t)$.

8) The acceleration of a particle moving along the x - axis at time $t > 0$ is given by $a(t) = \frac{1}{t^2}$. When $t = 1$ second, the particle is at $x = 2$ and has a velocity of -1 unit per second. If $x(t)$ is the particle's position, then what is the particle's position when $t = e$?

9) A particle moves along the x - axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

- Find the acceleration $a(t)$ of the particle at any time t .
- Find all values of t , $0 \leq t \leq 2$, for which the particle is at rest.
- Find the position $x(t)$ of the particle at any time t if $x(0) = 0$.

10) Given the acceleration, $a(t) = -4\sin 2t$, with initial velocity $v(0) = 2$, and the initial position of the body as $s(0) = -3$. Find the body's position at time t .

Answer Key:

1) $f(0) = 3$	2) $x(t) = 4\ln t + \frac{1}{3t^3} + 8t - \frac{13}{3}$
3) $f(x) = 2x^4 - 5x^2 + 5x + 3$	4) $f(x) = 6e^x - \sin x - 2$
5) $y = 9\tan x + \frac{2}{5}x^{1/2} + 5$	6) $y(2) = 30$
7) $v(t) = 6\sec t + 3t^2 - 3e^t + 2$	8) $x(e) = 1$
9) a) $a(t) = -2\pi\cos(2\pi t)$ b) $t = \frac{1}{4}, \frac{5}{4}$ c) $x(t) = t + \frac{\cos 2\pi t}{2\pi} - \frac{1}{2\pi}$	10) $s(t) = \sin(2t) - 3$

