Unit #8: Taylor Polynomials and Power Series
Topic: Power Series
Objective: SWBAT find the interval of convergence for a given power series.

Warm Up #3:

Use a third-degree Taylor approximation of e^x for x near 0 to find $\lim_{x \to 0} \frac{e^x - 1}{2x}$, then compare it to the actual limit at zero.

Now, what if we try to do the same thing to find $\lim_{x \to 0} \frac{sinx}{x}$.

Power Series

If x is a variable, then an infinite series of the form

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n + \cdots$$

is called a **power series.**

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \cdots a_n(x-c)^n + \cdots$$

is a power series **centered at c,** where a_0, a_1, a_2, ... and c are real numbers.

The major characteristic of a power series is that it involves a variable, and its convergence depends on the value(s) that x takes.

A power series may converge for some values of x and diverge for others.

Each power series has an ***interval of convergence***, that is, the set of values of x for which the power series converges.

For a power series centered at c, ONE of the following MUST be true:

1) The series converges only at c and $R\ is\ 0$.
   (ALL power series converge at their center!!)

2) The series converges for ALL x and $R\ is\ \infty$.

3) There exists an $R > 0$ such that the series converges for $|x - c| < R$ and diverges for $|x - c| > R$. The corresponding domain $[(c - R), (c + R)]$, is called the **interval of convergence** or the domain of the power series.

R is called the **radius of convergence of the power series**

Finding the Interval of Convergence

For a general power series, the interval of convergence is determined by using the **RATIO TEST**.

$$R = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

provided that it exists or is equal to ∞.

The **endpoints** of the interval must be checked individually using one of the tests for convergence.

Example #1: Find the values of x for which the series $\sum_{n=0}^{\infty}(3x)^n$. State the radius of convergence.

Example #2: Find the interval of convergence for the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^n}{n2^n}$.

Practice Problems: Find the radius and interval of convergence for each of the following power series.

1. $\sum_{n=0}^{\infty} \frac{(x-3)^n}{3^n}$

2. $\sum_{n=0}^{\infty} \frac{(2x-1)^n}{n!}$

3. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$

4. $\sum_{n=2}^{\infty} \frac{(5x)^n}{n^3}$

5. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

6. $\sum_{n=0}^{\infty} n!\,(x-3)^n$

7. $\sum_{n=0}^{\infty} \frac{(x-5)^{n+1}}{n+1}$

8) $\sum_{n=0}^{\infty} \frac{(x+1)^{n}}{3^{n}}$

Answer Key

1) $0 < x < 6\,;\ \ R = 3$	2) $Convergs\ for\ all\ x\,; R = \infty$
3) $-1 \le x < 5\,;\ R = 3$	4) $-\frac{1}{5} \le x \le \frac{1}{5}\ ;\ R = \frac{1}{5}$
5) $Converges\ for\ all\ x\,;\ R = \infty$	6) $Converges\ at\ x = 3\ only\,; R = 0$
7) $4 \le x < 6\,;\ R = 1$	8) $-4 < x < 2\ ;\ R = 3$