

Unit #9: Rates of Change and Particle Motion

Topic: Velocity and Rates of Changes

Objective: SWBAT will be able to interpret the derivative in order to model and explain particle motion analytically.

Warm Up #4:

A particle moves along the x - axis in such a way that its position at time t for $0 < t < 5$ is given by $s(t) = t^3 - 6t^2 + 9t - 2$.

- (a) Find the velocity at time t .

- (b) Find the acceleration at time t .

- (c) What are the value(s) of t for which the particle is at rest?

- (d) Is the speed increasing or decreasing at $t = 4$? Explain your answer.

We have already looked at particle motion graphically, now let's look at what happens when we try to analyze motion by using the position equation.

Analytically:

- 1) A particle moves along the x -axis so that at time $t \geq 0$ hours, its position is given by $x(t) = t^3 - 3t^2 - 9t + 2$ miles.
- (a) At $t = 0$, is the particle moving to the right or to the left? Justify.
- (b) At what time(s) does the particle change directions. Justify.
- (c) At $t = 1/2$, is the velocity of the particle increasing or decreasing? Explain your answer.
- (d) At $t = 1/2$, is the speed of the particle increasing or decreasing? Explain your answer.
- (e) Find all values of t for which the particle is moving to the left.
- (f) What is the particle's acceleration at $t = \frac{1}{3}$? Explain, with units, the meaning of your answer in terms of the particle's velocity.

- 2) A particle moves along a horizontal line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in feet and t is measured in seconds.
- (a) Find the displacement of the particle during the first 2 seconds. Explain your answer.
 - (b) Find the average velocity of the particle during the first 4 seconds.
 - (c) Find the velocity of the particle when $t = 4$ seconds. Explain the meaning in terms of the particle's movement.
 - (d) Find the acceleration of the particle when $t = 4$ seconds. Explain its meaning in terms of the particle's velocity.
 - (e) At $t = 4$ seconds, is the speed of the particle increasing or decreasing? Justify.
 - (f) During what times is the particle moving to the right? left? At what values of t does the particle change direction? Justify your answers.
 - (g) Find the total distance the particle travels during the first 4 seconds.

Warm Up #5: CALCULATOR ALLOWED!

A particle moves along the x -axis with its position at time t given by

$$x(t) = \frac{t}{1+t^2}, t \geq 0$$

where t is measured in seconds and x in meters.

- (a) Find the velocity at time t and at $t = 2$ seconds.

- (b) When is the particle at rest? When is moving to the right? To the left? Justify your answers.

- (c) Find the displacement of the particle during the time interval $0 \leq t \leq 4$.

- (d) Find the total distance traveled by the particle during the time interval $0 \leq t \leq 4$.

- (e) Find the acceleration of the particle at time t and at $t = 3$ seconds.

- (f) When is the particle speeding up? Slowing down? Justify your answers.

3) A particle moves along the x - axis so that at time $t, t \geq 0$, its position is given by:

$$x(t) = 2t^3 - 9t^2 + 12t - 4$$

- (a) Find all t for which the distance x is increasing.

- (b) Find all t for which the velocity is increasing.

- (c) Find all t for which the speed of the particle is increasing.

- (d) Find the speed when $t = \frac{3}{2}$.

- (e) Find the total distance traveled on the time interval $[0,4]$.

- 4) A particle moves along a horizontal line so that its position at any time is given by $s(t) = t^3 - 12t^2 + 36t$, $t \geq 0$, where s is measured in meters and t in seconds.
- (a) Find the instantaneous velocity at time t and at $t = 3$ seconds.
- (b) When is the particle at rest? Moving to the right? Moving to the left? Justify your answers.
- (c) Find the displacement of the particle after the first 8 seconds.
- (d) Find the total distance traveled by the particle during the first 8 seconds.
- (e) Find the acceleration of the particle at time t and at $t = 3$ seconds.
- (f) When is the particle speeding up? Slowing down? Justify your answers.

- 5) A particle moves along a horizontal line so that its position at any time $t \geq 0$ is given by $s(t) = -t^3 + 7t^2 - 14t + 8$ is measured in meters and t in seconds.
- (a) Find the instantaneous velocity at any time t and when $t = 2$.
 - (b) Find the acceleration of the particle at any time t and when $t = 2$.
 - (c) When is the particle at rest? When is moving to the right? To the left? Justify your answers.
 - (d) Find the displacement of the particle during the first two seconds?
 - (e) Find the total distance traveled by the particle during the first two seconds?
 - (f) Are the answers to (d) and (e) the same? Explain.
 - (g) When is the particle speeding up? Slowing down? Justify your answers.