

Unit #10: Applications of Differentiation

Topic: First Derivative Test

Objective: SWBAT identify where a function is increasing/decreasing by using the first derivative.

## Warm Up #4:

**CALCULATOR ALLOWED!**

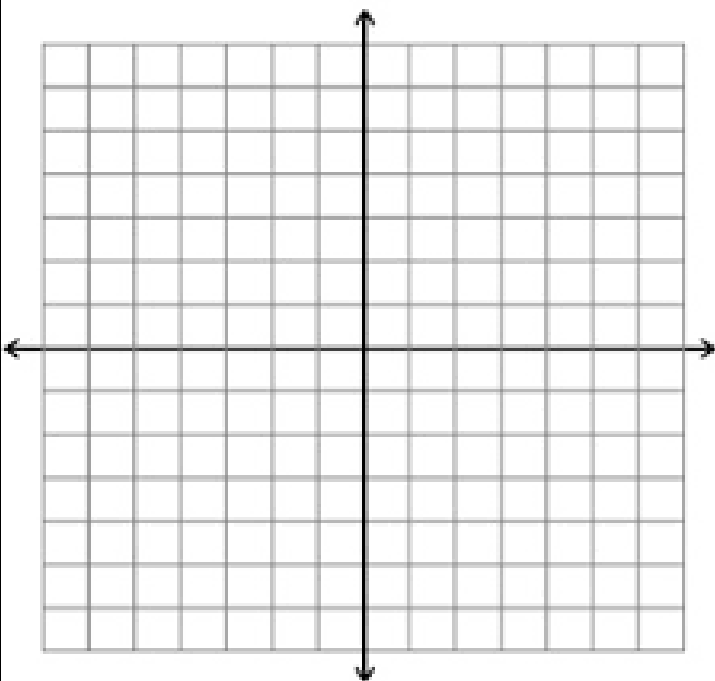
Graph the derivative of  $f(x)$  and then use the graph to identify:

(a) Where  $f(x)$  is increasing? decreasing?  
Explain your reasoning.

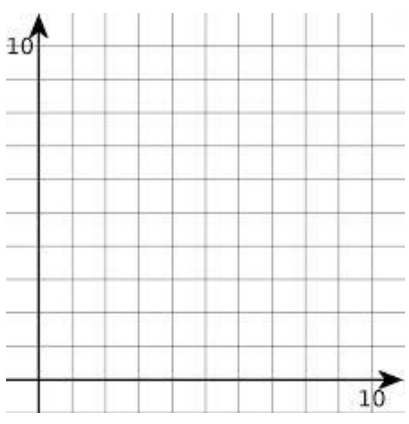
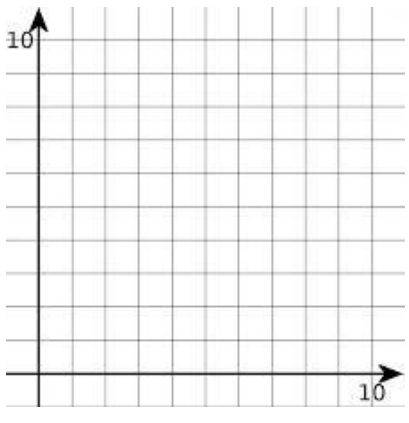
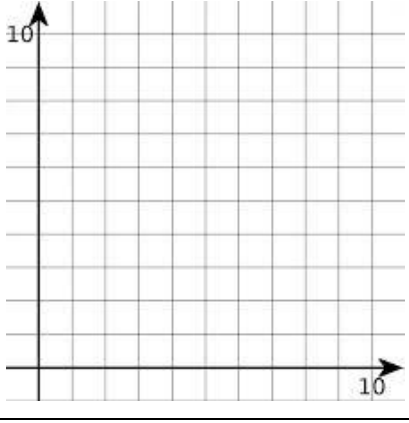
(b) Does  $f(x)$  have any horizontal tangent lines? Explain your reasoning.

(c) Does  $f(x)$  have any minimum points? maximum points? Justify your answers.

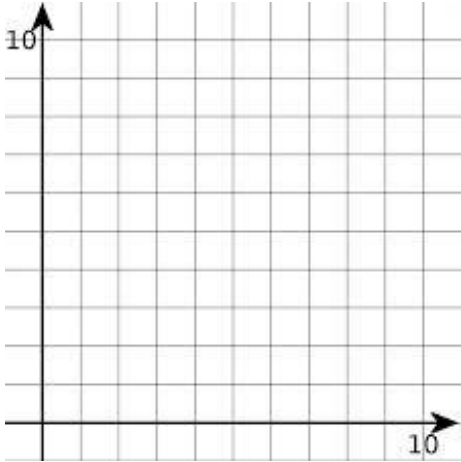
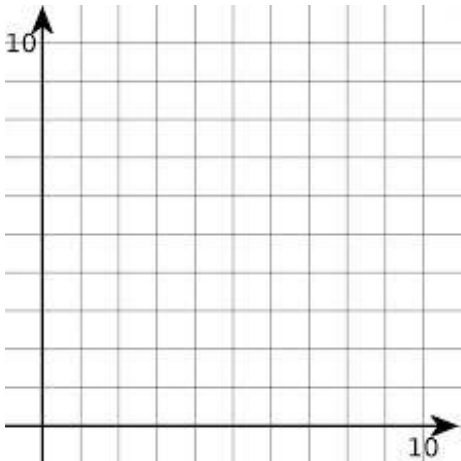
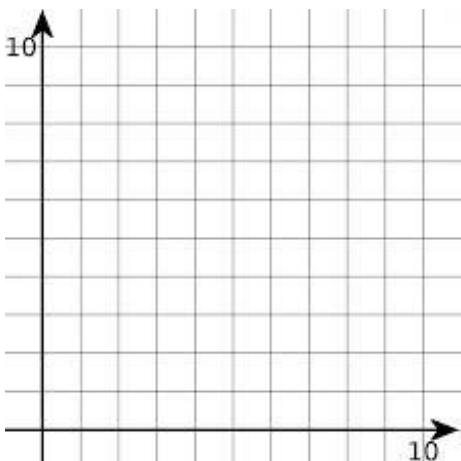
$$f(x) = (x^2 - 3)(x^2 + 2x); x \leq 2$$



### Increasing, Decreasing, and the 1<sup>st</sup> Derivative Test

<i>What does it sound like??</i>	<i>What does it look like??</i>
If _____, then  $f$ is _____.	
If _____, then  $f$ is _____.	
If _____, then  $f$ is _____.	

*At what values of  $x$  can the graph of a function change its increasing/decreasing/constant status??*

<i>What does it sound like??</i>	<i>What does it look like??</i>
<p>If <math>f'(x)</math> changes from _____ at <math>x = c</math>, then <math>f</math> has a _____ at <math>x = c</math>.</p>	
<p>If <math>f'(x)</math> changes from _____ at <math>x = c</math>, then <math>f</math> has a _____ at <math>x = c</math>.</p>	
<p>If <math>f'(x)</math> _____ change signs at <math>x = c</math>, then <math>f</math> _____ have a min/max at <math>x = c</math>.</p>	

**When using the First Derivative Test, you MUST write a concluding statement that indicates the type of sign change at  $x = c$ .**

*Practice Problems:* Use the first derivative test to identify where the function is increasing/decreasing/constant and where the relative extrema are. Justify your answers.

1)  $f(x) = x \left( 4 + x^2 - \frac{x^4}{5} \right)$  **CALCULATOR ALLOWED!**

2)  $y = (x^2 - 4)^{2/3}$

3)  $g(x) = 1 + x^2 - 2x^4$

$$4) h(x) = x\sqrt{8 - x^2}$$

$$5) f(x) = -2x^3 + 6x^2 - 3$$

$$6) y = 2x^4 - 4x^2 + 1$$

$$7) g(x) = x^4 - \frac{2}{3}x^3$$

$$8) y = 3x^4 - 4x^3 - 12x^2 + 5$$

**Answer Key:**

1) decr:  $(-\infty, -2)(2, \infty)$ ; incr:  $(-2, 2)$ ; rel min at  $x = -2$ ; rel max at  $x = 2$

2) decr:  $(-\infty, -2)(0, 2)$ ; incr:  $(-2, 0)(2, \infty)$ ; rel min at  $x = \pm 2$ ; rel max at  $x = 0$

3) decr:  $(-\frac{1}{2}, 0)(\frac{1}{2}, \infty)$ ; incr:  $(-\infty, -\frac{1}{2})(0, \frac{1}{2})$ ; rel min at  $x = 0$ ; rel max at  $x = \pm \frac{1}{2}$

4) decr:  $(-\sqrt{8}, -2)(2, \sqrt{8})$ ; incr:  $(-2, 2)$ ; rel min at  $x = -2$ ; rel max at  $x = 2$

5) decr:  $(-\infty, 0)(2, \infty)$ ; incr:  $(0, 2)$ ; rel min at  $x = 0$ ; rel max at  $x = 2$

6) decr:  $(-\infty, -1)(0, 1)$ ; incr:  $(-1, 0)(1, \infty)$ ; rel min at  $x = \pm 1$ ; rel max at  $x = 0$

7) decr:  $(0, \frac{1}{2})$ ; incr:  $(\frac{1}{2}, \infty)$ ; rel min at  $x = \frac{1}{2}$

8) decr:  $(-\infty, -1)(0, 2)$ ; incr:  $(-1, 0)(2, \infty)$ ; rel min at  $x = -1$  and  $2$ ; rel max at  $x = 0$

