Unit #1: Integration Review
Topic: The Fundamental Theorem of Calculus
Objective: *SWBAT use the Fundamental Theorem of Calculus to integrate a function.*
SWBAT use the Fundamental Theorem of Calculus to find the derivative of an Integral.

Warm Up #4:

Find the average value of $f(x) = (x-3)^2$ over $[2, 6]$.

The *Fundamental Theorem of Calculus* (FTC) is considered the most important computational study in the history of mathematics. It describes the fundamental relationship between differentiation and integration.

Part 1:

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Part 2:

If f is continuous on $[a, b]$, then the function $F(x) = \int_a^x f(t)dt$ has a derivative at every point in $[a, b]$ and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

This is the derivative of an integral where the lower limit is a **constant** and the derivative matches upper limit of integration.

***Problem Set #4**: Evaluate each of the following and then identify which part of the FTC is being used.*

1) $\int_1^3 (2x^2 - 12x + 13)dx$	2) $\frac{d}{dx}\left[\int_2^x \sqrt{1+t^2}\, dt\right]$
3) $\int_{-4}^{-1} -\frac{4}{x^3} dx$	4) $\int_{1/2}^{3} (2 - \frac{1}{x})dx$
5) $\frac{d}{dx}\left[\int_{-1}^{x^2} (-2t + 2)dt\right]$	6) $\frac{d}{dx}\left[\int_{2x}^{1} (6cos^2 t)dt\right]$

7) $\int_{-1}^{1} e^{2x-2}dx$	8) $\int_{-\pi/4}^{-\pi/6} 2cosxdx$
9) $\int_{-2}^{-1} \frac{2}{x^2}dx$	10) $\frac{d}{dx}\left[\int_{6}^{x^2} cot3t\ dt\right]$
11) $\frac{d}{dx}\left[\int_{3x}^{6} ln(1+t^2)dt\right]$	12) $\int_{0}^{4} \frac{1-\sqrt{u}}{\sqrt{u}}du$

13) $\int_{-1}^{ln2}(e^x - 1)dx$	14) $\frac{d}{dx}\left[\int_2^{5x}\frac{\sqrt{1+u^2}}{u}\,du\right]$
15) $\frac{d}{dx}\left[\int_2^x(3t + cos^2t)\,dt\right]$	16) $\int_0^3\frac{dx}{4x+3}$

Answer Key:

1) $-\frac{14}{3}$	2) $\sqrt{1+x^2}$	3) $\frac{15}{8}$	4) $5 - ln6$
5) $-4x^3 + 4x$	6) $-12cos^2(2x)$	7) $\frac{1}{2} - \frac{1}{2e^4}$	8) $-1 + \sqrt{2}$
9) 1	10) $2xcot3x^2$	11) $-3ln(1 + 9x^2)$	12) 0
13) $1 - ln2 - \frac{1}{e}$	14) $\frac{\sqrt{1+25x^2}}{x}$	15) $x + cos^2x$	16) $\frac{1}{4}ln5$