

## Unit #5: Limits

Topic: Properties of Limits

Objective: SWBAT find limits by applying the properties of limits.

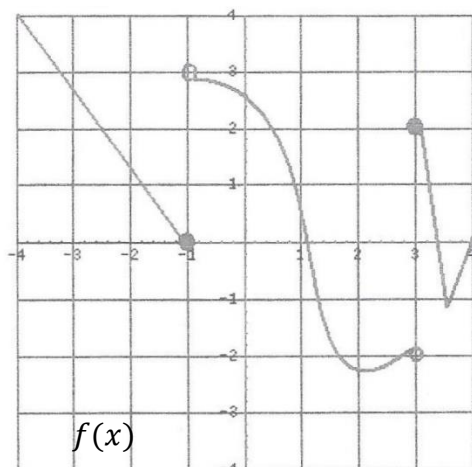
**Warm Up #4:**

Find each of the following limits based on the given function.

a)  $\lim_{x \rightarrow -2} \frac{x^2 - 4x}{x - 2} =$

b)  $\lim_{x \rightarrow 1} \frac{3x^2 - 3x}{x - 1} =$

c)



$$\lim_{x \rightarrow -1} f(x) =$$

$$d) f(x) = \begin{cases} x^2 & \text{for } x < -1 \\ \frac{x+6}{x^2-x} & \text{for } -1 < x < 2 \\ 3x - 2 & \text{for } x \geq 2 \end{cases}$$

$$\text{Find } \lim_{x \rightarrow -1} f(x) =$$

$$\text{Find } \lim_{x \rightarrow 2} f(x) =$$

## Properties of Limits

There are six basic facts about limits that can be used to calculate unfamiliar limits from limits we already know.

**LIMIT LAWS** Suppose that  $c$  is a constant and the following limits exist.

$$\lim_{x \rightarrow a} f(x) \quad \text{and} \quad \lim_{x \rightarrow a} g(x)$$

Then

$$1. \lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

$$2. \lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

$$3. \lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$$

$$4. \lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$5. \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \quad \text{if } \lim_{x \rightarrow a} g(x) \neq 0$$

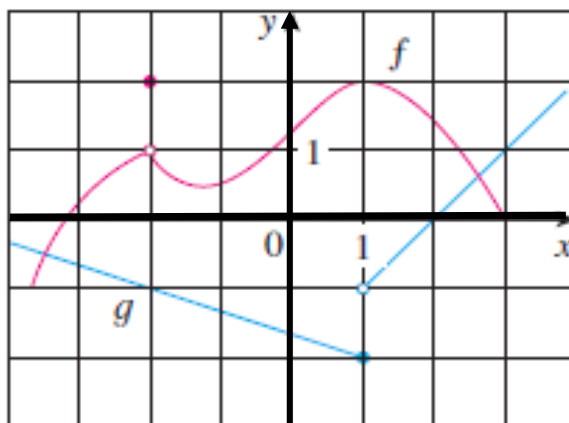
$$6. \lim_{x \rightarrow a} c = c$$

*Example #1:* Use the limit laws to evaluate each of the following limits, if they exist.

If  $\lim_{x \rightarrow 4} f(x) = 0$  and  $\lim_{x \rightarrow 4} g(x) = 3$ , find

a) $\lim_{x \rightarrow 4} (2f(x) + 5g(x))$	b) $\lim_{x \rightarrow 4} (g(x) + 3)$
c) $\lim_{x \rightarrow 4} [-3f(x)g(x)]$	d) $\lim_{x \rightarrow 4} \frac{g(x)}{f(x) - 1}$

Example #2:



Use the limit laws and the graphs of  $f$  and  $g$  above to evaluate the following limits, if the exist.

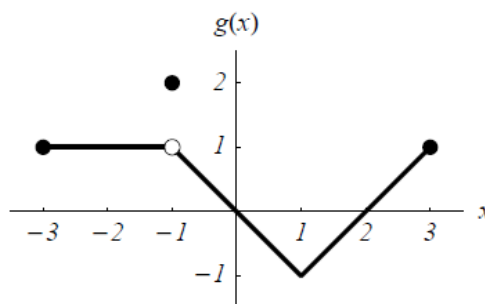
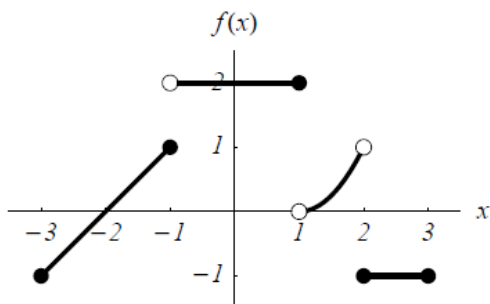
a) $\lim_{x \rightarrow -2} [f(x) + 5g(x)]$	b) $\lim_{x \rightarrow 1} [f(x)g(x)]$	c) $\lim_{x \rightarrow 2} \frac{f(x)}{[3g(x) + 1]}$
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*Let's Practice:* Use the limit laws and any graphs given to find each of the following limits or state that the limit does not exist.

1) If  $\lim_{x \rightarrow c} f(x) = 5$  and  $\lim_{x \rightarrow c} g(x) = -3$ , find

a) $\lim_{x \rightarrow c} [-2g(x)]$	b) $\lim_{x \rightarrow c} [\frac{1}{2}f(x) + g(x)]$
c) $\lim_{x \rightarrow c} \frac{f(x) + 2}{g(x) + 3}$	d) $\lim_{x \rightarrow c} \sqrt{f(x)}$

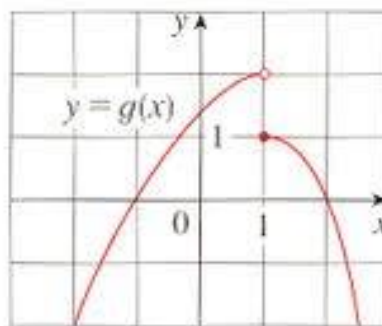
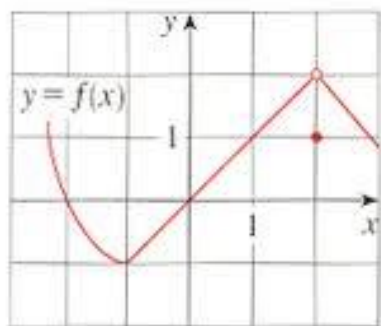
2)



a)  $\lim_{x \rightarrow 0} [2f(x) - g(x)]$

b)  $\lim_{x \rightarrow -1} [f(x)g(x)]$

3)



a)  $\lim_{x \rightarrow -1} [6f(x) + g(x)]^2$

b)  $\lim_{x \rightarrow 1} \left[ \frac{f(x)}{g(x)} \right]$

4) If  $\lim_{x \rightarrow c} f(x) = \frac{3}{2}$  and  $\lim_{x \rightarrow c} g(x) = -\frac{1}{2}$ , find

a)  $\lim_{x \rightarrow c} (f(x) + g(x))^2$

b)  $\lim_{x \rightarrow c} (6f(x)g(x))$

c)  $\lim_{x \rightarrow c} \frac{5g(x)}{4f(x)}$

d)  $\lim_{x \rightarrow c} \frac{1}{\sqrt{f(x)}}$

Name \_\_\_\_\_ Date \_\_\_\_\_

## Homework - Properties of Limits

1. Given that

$$\lim_{x \rightarrow 2} f(x) = 4 \quad \lim_{x \rightarrow 2} g(x) = -2 \quad \lim_{x \rightarrow 2} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why.

(a)  $\lim_{x \rightarrow 2} [f(x) + 5g(x)]$

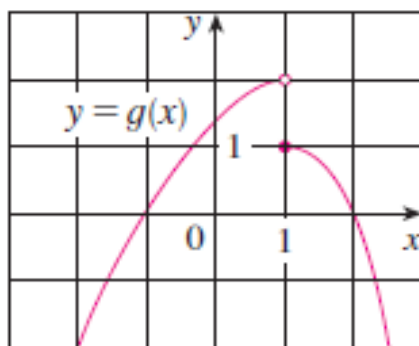
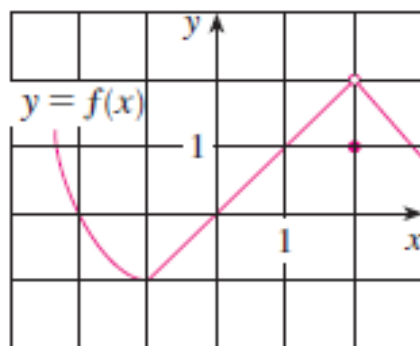
(b)  $\lim_{x \rightarrow 2} [g(x)]^3$

(c)  $\lim_{x \rightarrow 2} \sqrt{f(x)}$

(d)  $\lim_{x \rightarrow 2} \frac{3f(x)}{g(x)}$

(e)  $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$

(f)  $\lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)}$

2. The graphs of  $f$  and  $g$  are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.

(a)  $\lim_{x \rightarrow 2} [f(x) + g(x)]$

(b)  $\lim_{x \rightarrow 1} [f(x) + g(x)]$

(c)  $\lim_{x \rightarrow 0} [f(x)g(x)]$

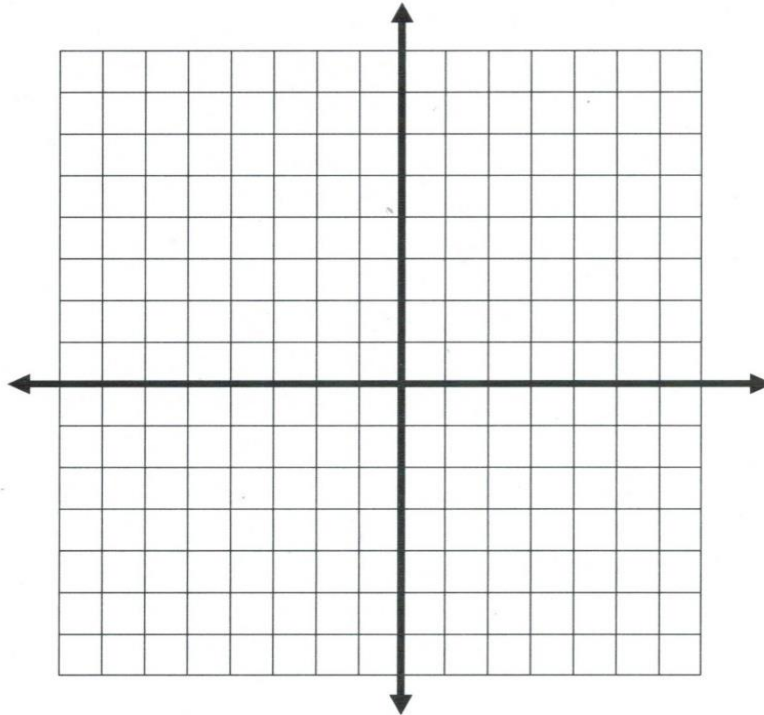
(d)  $\lim_{x \rightarrow -1} \frac{f(x)}{g(x)}$

(e)  $\lim_{x \rightarrow 2} [x^3 f(x)]$

(f)  $\lim_{x \rightarrow 1} \sqrt{3 + f(x)}$

3) Sketch piecewise function below and then use the graph to identify the following limits, if they exist.

$$f(x) = \begin{cases} |2x + 4| & \text{for } x < 1 \\ 2 & \text{for } 1 \leq x < 3 \\ -x + 5 & \text{for } x \geq 3 \end{cases}$$



a)  $\lim_{x \rightarrow 0} \frac{f(x)}{2} =$

b)  $\lim_{x \rightarrow 1} f(x) =$

c)  $\lim_{x \rightarrow 3} f(x) =$

d)  $f(1) =$

e)  $f(3)$