Unit #8: Taylor Polynomials and Power Series
Topic: Taylor and Maclaurin Series
Objective: SWBAT manipulate various Taylor and Maclaurin series.

Warm Up #4:

Find the first four nonzero terms for the Taylor series $f(x) = e^{5x}$ centered at $c = 2$.

Taylor and Maclaurin Series

We will now look at a special family of power series that we are already acquainted with.

Taylor Series centered at $x = c$:

$$\boldsymbol{f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots \frac{f^n(c)}{n!}(x-c)^n + \cdots = \sum_{n=0}^{\infty} \frac{f^n(c)}{n!}(x-c)^n}$$

Once again, if $c = 0$ the series is called a **Maclaurin Series**.

Notice that now we use an equal sign instead of an approximation sign

These series can be easily manipulated to suit other similar functions.

1) Substitute into a series for x
2) Multiply or divide the series by a constant and/or a variable
3) Add or subtract two series
4) Differentiate or integrate a series (may change the interval of convergence, but not the radius of convergence)
5) Recognize the series as the sum of a geometric power series

Simple Substitutions:

Example #1: Find the first four nonzero terms and the general term for the Taylor series $f(x) = e^{5x}$ centered at $c = 2$. (HINT: Refer back to the WARM UP)

Example #2: Find a Maclaurin Series for $f(x) = \frac{cos(3x)}{x}$. Give the first four nonzero terms and the general term.

Practice Problems: Find the first four nonzero terms and the general term for each of the following Taylor series using a series that you already know.

1) $f(x) = \ln(x+1)$

2) $f(x) = e^{x/2}$

3) $g(x) = sin\left(\frac{1}{2}x\right)$

4) $f(x) = \frac{1}{2-x}$ centered at $c = 1$

5) $g(x) = \frac{ln(1+x)}{x}$

6) $f(x) = 1 - e^{-2x}$

7) $f(x) = \frac{e^x + e^{-x}}{2}$

8) $g(x) = \frac{3x}{1-x^2}$

9) $h(x) = x cos\sqrt{x}$

Answer Key

1) $f(x) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots \frac{(-1)^n x^{4n}}{(2n)!} + \cdots$
2) $f(x) = x + \frac{x}{2} + \frac{x^2}{2^2 \cdot 2!} + \frac{x^3}{2^3 \cdot 3!} + \cdots \frac{x^n}{2^n \cdot n!} + \cdots$
3) $f(x) = \frac{x}{2} - \frac{x^3}{2^3 \cdot 3!} + \frac{x^5}{2^5 \cdot 5!} - \frac{x^7}{2^7 \cdot 7!} + \cdots \frac{(-1)^n x^{2n+1}}{2^{2n+1}(2n+1)!} + \cdots$
4) $f(x) = 1 + (x-1) + (x-1)^2 + (x-1)^3 + \cdots (x-1)^n + \cdots$
5) $f(x) = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \cdots \frac{(-1)^{n+1} x^{n-1}}{n} + \cdots$
6) $f(x) = 2x - \frac{2^2 x^2}{2!} + \frac{2^3 x^3}{3!} - \frac{2^4 x^4}{4!} + \cdots \frac{(-1)^n (2x)^{n+1}}{(n+1)!} + \cdots$
7) $f(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots \frac{x^{2n}}{(2n)!} + \cdots$
8) $f(x) = 3x + 3x^3 + 3x^5 + 3x^7 + \cdots 3x^{2n+1} + \cdots$
9) $f(x) = x - \frac{x^2}{2!} + \frac{x^3}{4!} - \frac{x^4}{6!} + \cdots \frac{(-1)^n x^{n+1}}{(2n)!} + \cdots$