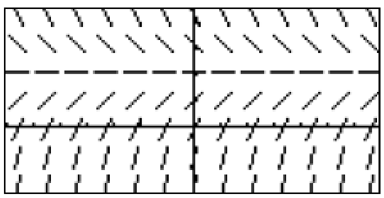
Unit #3: Differential Equations
Topic: Euler's Method
Objective: SWBAT estimate a particular solution to a differential equation by using Euler's Method.

Warm Up #5:

Match the slope fields with their differential equations.

(A)



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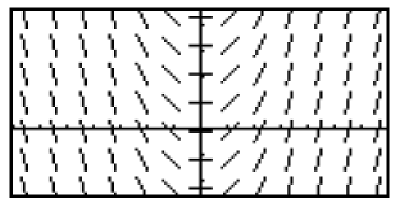

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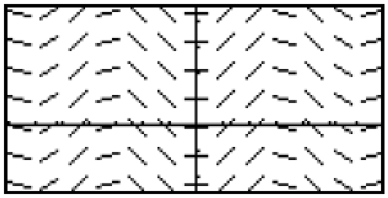

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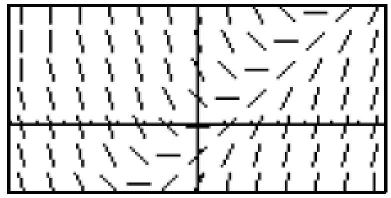


7. $\dfrac{dy}{dx} = \sin x$

8. $\dfrac{dy}{dx} = x - y$

9. $\dfrac{dy}{dx} = 2 - y$

10. $\dfrac{dy}{dx} = x$

Euler's Method:

In order to solve differential equations, we have different methods:

Graphically – Slope fields

Analytically – Separable differential equations

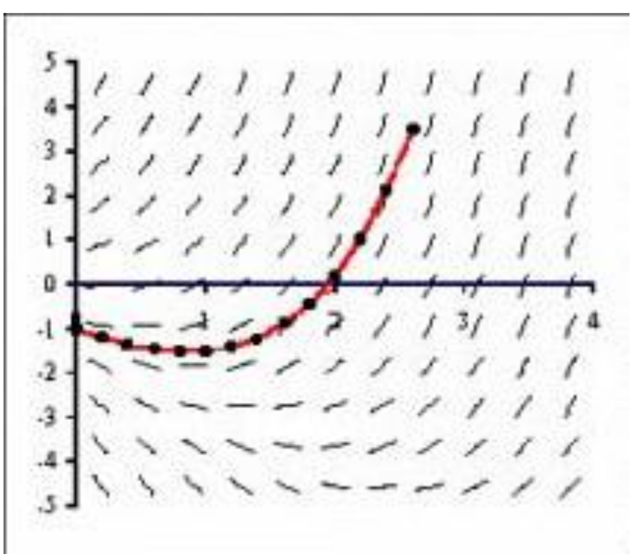
And now …

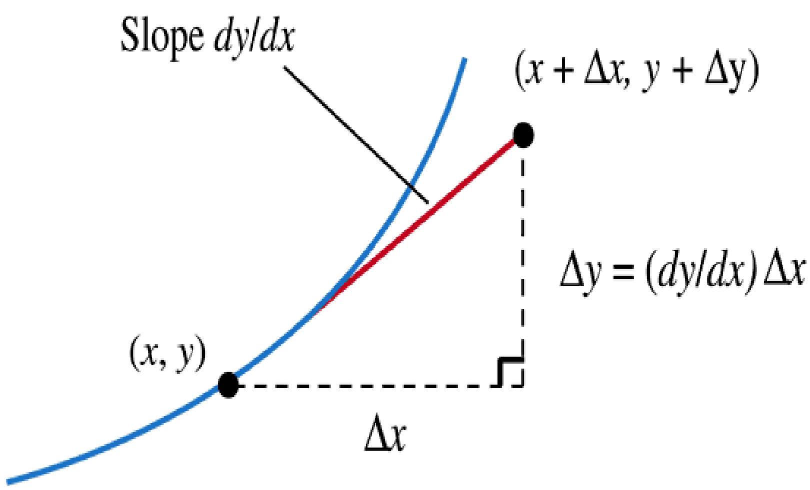
Numerically – Euler's method

There are many differential equations that cannot be solved easily, but we can still find an approximate solution.

Euler's Method gives us a numerical approach to finding an estimation for a particular solution to a differential equation by using a series of successive linear approximations.

Essentially we will be following tangent lines in a slope field to show the solution curve. The further away you get the less accurate your approximation will be.





Example #1: ***NON-CALCULATOR***

Consider the differential equation $\frac{dy}{dx} = x + y$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 0$.

Use Euler's Method with 4 steps of equal size, starting at $x = 2$, to approximate $f(3)$. Show the work that leads to your answer.

Example #2: ***CALCULATOR***

If $\frac{dy}{dx} = 2x - y$ and if $y = 3\ when\ x = 2$, use Euler's method with five equal steps to approximate $y\ when\ x = 1.5$.

Problem Set #5:

1) (a) Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using Euler's method with two equal steps.

(b) Solve the differential equation $\frac{dy}{dx} = x + 2$ with the initial condition $y(0) = 3$, and use your solution to find $y(1)$.

(c) The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

2) The curve passing through (2, 0) satisfies the differential equation $\frac{dy}{dx} = 4x + y$. Find an approximation to $y(3)$ using Euler's Method with two equal steps.

3) Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show work that leads to your answer.

(b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

4) Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(0) = -1$.

(a) Use Euler's method with two steps of equal size, starting at $x = 0$, to approximate $f(\frac{1}{2})$. Show work that leads to your answer.

(b) Find $y = f(x)$, the particular solution to the given differential equation with the initial condition $f(0) = -1$.

Warm Up #6:

Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.6)$.

Problem Set #6:

5) Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x, with $f(1) = 4$.

(a) Use Euler's Method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.

(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = -3xy$ with the initial condition $f(1) = 4$.

6) Consider the differential equation $\frac{dy}{dx} = \frac{y}{2}cos(\pi x)$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(0) = 2$.

(a) Use Euler's method with four steps of equal size, starting at $x = 0$, to approximate $f(2)$. Show work that leads to your answer.

(b) Find $y = f(x)$, the particular solution to the given differential equation with the initial condition $f(0) = 2$ and then evaluate $f(2)$.

7) Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.

(a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

(b) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.

8) The table gives selected values for the derivative of a function f on the interval $-2 \le x \le 2$. If $f(-2) = 3$ and Euler's method with a step-size of 1.5 is used to approximate $f(1)$, what is the resulting approximation?

x	$f'(x)$
-2	-0.8
-1.5	-0.5
-1	-0.2
-0.5	0.4
0	0.9
0.5	1.6
1	2.2
1.5	3
2	3.7

Answer Key:

1) a) $\frac{21}{4}$ b) $\frac{11}{2}$ c) $\frac{1}{4}$	2) 11	3) a) $\frac{17}{4}$
3) b) $y = 6 - 4e^{-\frac{1}{3}(x^3+1)}$	4) a) -11/32 b) $y = -\frac{1}{x^2+2x+1}$	
5) a) 2.5 b) $y = 4e^{-\frac{3}{2}(x^2+1)}$	6) a) 15/8 b) $y = 2e^{\frac{1}{2\pi}sin\pi x}$, 2	
7) a) -5/4 b) 1/3 c) $y = 1 - e^{1-x}$	8) 2.4	