Unit #2: Logarithms
Topic: Exponential Growth and Decay
Objective: SWBAT solve exponential growth and decay problems by using logarithms.

Warm Up #5:

CALCULATOR ALLOWED

You deposit $1600 in a bank account. Find the balance after 3 years for each of the following situations:

    a) The account pays 2.5% interest compounded monthly.

    b) The account pays 4% interest compounded continuously.

Exponential Growth and Decay:

A specific type of exponential growth/decay model will occur when a quantity that increases or decreases exponentially grows or decays, respectfully, at a rate proportional to the current amount.

Stated a different way:

    The more you _____, the more you _____ or, the _____

                    you have, the less you _____.

Things that grow or decay exponentially tend to increase or decrease by a _____

_____ for each cycle usually known as _____.

The general formula that will be used to represent this exponential model is:

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The key to solving these types of problems usually involves determining _____.

For real-life applications, our independent variable is usually _____.

Example #1:

A bacteria culture starts with 500 bacteria and after 3 hours there are 8000 bacteria (assume exponential growth model).

a) Find an expression for the number of bacteria after t hours.

b) Find the number of bacteria after 4 hours.

c) When will the population reach 30,000?

Many elements are unstable as are many isotopes of stable elements. These elements and isotopes undergo radioactive decay, losing a fixed fraction of their mass per unit of time. The time it takes for these samples to lose half of their mass is called the half-life of the substance.

Example #2:

The radioactive isotope Cobalt-60 (Co-60) has a half-life of 5.24 years. You have an initial amount of 80 grams.

a) Write the amount as a function of time (in years).

b) How much Co-60 is left after 2 years?

c) How much time has passed when 40% of the initial amount is left?

Problem Set #5: Read each problem carefully and show all work.

1) A population of bacteria is growing exponentially. It takes the population 15 minutes to double. How long will it take for the population to triple (3)?.

2) The population of a certain country is growing exponentially according to the model $P = P_0 e^{rt}$ where P is the population in millions and t is the number of years after 1980. In 1988 the population was 35 million, and in 1995 the population was 48 million. What is the growth rate r?

3) The number of bacteria in a certain population increases according to a continuous exponential growth model, with a growth rate of 8.4% per hour. How many hours does it take for the size of the sample to double?

4) A population of bacteria doubles after 10 hours. Assume it grows exponentially. Find its relative growth rate.

5) The half-life of a radioactive isotope describes the amount of time that it takes half of the isotope in a sample to decay. In the case of radiocarbon dating, the half-life of carbon 14 is 5,730 years. A fossil is found that has 35% carbon 14 compared to the living sample. How old is the fossil?

6) A colony of insects doubles every 10 days. If the colony has 850 insects today, how many are/were present

   a) in 30 days?
   b) 10 days ago?

Warm Up #6:

A population of a small city had 3000 people in the year 2000 and has grown at a rate proportional to its size. In the year 2005 the population was 3700.

a) Find an expression for the number of people in the city t years after the year 2000.

b) Estimate the population of the city in 2006. In 2010.

c) Find the rate of growth of the population in 2006.

d) Assuming the growth continues at the same rate, when will the town have 25000 people?

Problem Set #6: Read each problem carefully and show all work.

7) A cup of coffee contains approximately 96 mg of caffeine. When you drink the coffee, the caffeine is absorbed into the bloodstream and is eventually metabolized by the body. Every 5 hours the amount of caffeine present in the body is reduced by one-half. How many hours does it take for the amount of caffeine to be reduced to 12 mg?

8) A bacterium grows exponentially with a constant relative growth rate. After 2 hours there are 600 bacteria and after 8 hours the count is 75,000. How many bacterium were there initially?

9) A bacteria culture starts with 3,000 bacteria. After 3 hours there are 48,000 bacteria present. How long did it take the original amount to double?

10) Several fruit flies have found their way into your kitchen and reproduce exponentially. You first count only 5, but after 2 days you find 15. After how many days can you expect to be overwhelmed by a swarm of 100 flies?

11) At time $t = 0$ you have 100 grams of radioactive Chromium-48. Ten hours later you have 74 grams left. What is the half life of Chromium-48?

12) Ra-225, Radium 225, a radioactive isotope of Ra-226, has a half-life of approximately 15 days. Ra-226 is the most stable atomic weight, but still radioactive, with a half-life of 1600 years. If we start with a 6 gram sample of Ra-225,

a) how much of the original sample is left after 12 weeks?
b) how many days will it take the original sample to decay to 1 gram?

Answer Key:

1) 23.774 min	2) 0.045
3) 8.252 hrs	4) $k = \frac{\ln 2}{10}$
5) 8678.504 yrs. old	6) a) 6800 b) 425
7) 15	8) 120
9) 0.75 hr	10) 5.454 days
11) 23.020 hrs	12) a) 0.124 grams b) 38.774 days