

Unit #11: Related Rates

Topic: More Related Rates Problems

Objective: SWBAT apply derivatives to real life applications.

Warm Up #5:

The radius of a circle is increasing at a constant rate of 0.4 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference is 60π ?

Multi-Formula Problems

Sometimes to solve a related rates problem we need to use more than one formula.

Example #1:

A beach ball is deflating at a constant rate of 10 cubic centimeters per second. When the volume of the ball is $\frac{256}{3}\pi$ cubic centimeters, what is the rate of change of the surface area? ($S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$)

Problem Set #5: Read each question carefully and show ALL work.

- 1) The radius of a cylinder is increasing at a rate of 1 unit per minute, while the height of the cylinder is decreasing at the rate of 2 units per minute. When the radius is 3 units and the height is 5 units, is the volume of the cylinder increasing or decreasing, and how fast? What is the rate of change of the surface area?

- 2) Gas is escaping from a spherical balloon at the rate of $2 \text{ ft}^3/\text{min}$. How fast is the surface area changing when the radius is 12 ft ?

- 3) The volume of a cube is changing at the constant rate of $75 \text{ cubic cm}/\text{min}$.
- a) Find the rate of change of an edge of the cube when the length of the edge is 5 cm .
- b) Find the rate of change of the surface area when the surface area is 54 cm^2 .
- 4) If the volume of a cube is increasing at $24 \text{ in}^3/\text{min}$ and the surface area of the cube is increasing at $12 \text{ in}^2/\text{min}$, what is the length of each edge of the cube?

- 5) A spherical balloon is inflated with helium at the rate of $100\pi \text{ ft}^3/\text{min}$.
- How fast is the balloon's radius increasing at the instant the radius is 5 ft ?
 - How fast is the surface area increasing at that instant?
- 6) The edge of a cube is expanding at a rate of $1.25 \text{ cm}/\text{sec}$.
- How fast is the volume changing when the length of an edge is 12 cm ?
 - How fast is the surface area changing at the same instant?
 - What is the length of the edges of the cube at the instant the rate of change of the volume is equal to the rate of change of the surface area?

Warm Up #6:

More Related Rates Problems

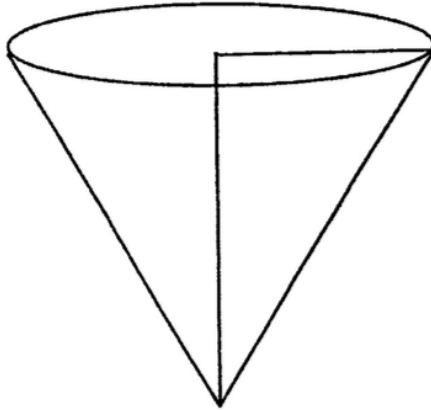
Sand is pouring from a pipe at a rate of $16 \text{ ft}^3/\text{sec}$. The falling sand forms a conical pile on the ground. The altitude of the pile is always $\frac{1}{4}$ the diameter of the base.

$$\left(\text{Remember: } V_{\text{cone}} = \frac{1}{3}\pi r^2 h \right)$$

- What is the rate of change of the altitude at the instant the altitude is 4 ft ?
- What is the rate of change of the height of the pile at the instant that area of the base of the pile is $36\pi \text{ ft}^2$?
- What is the height of the pile at the instant the rate of change of the volume is equal to the rate of change of the area of the base?
- What is the height of the pile when the rate of change of the volume is equal to the rate of change of the height of the pile?

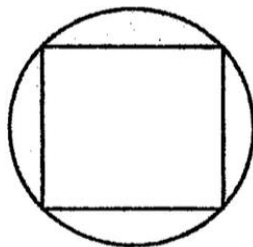
Problem Set #6: Read each question carefully and show ALL work.

- 7) Water is poured into a conical tank that is 24 feet tall and has a diameter at the top of 20 feet. ($V = \frac{1}{3}\pi r^2 h$)



- (a) Write the formula for the volume of the cone of water in terms of h , the height of the water in the tank.
- (b) When the volume of the water is increasing at 3.4 cubic feet per minute and the height of the water is 2 feet, at what rate is the height of the water changing?
- (c) The radius of the surface of the water in the tank is increasing at 0.75 feet per minute. At what rate is the area of the surface changing when the radius is 4.2 feet?

- 8) A square is inscribed in a circle. The radius of the circle is increasing at a constant rate of 0.8 centimeters per second.



- (a) When the side of the square is 4 centimeters, what is the area of the circle? Include units.
- (b) When the side of the square is 4 centimeters, what is the rate of change in the area of the circle? Include units.
- (c) When the radius of the circle is $5\sqrt{2}$ centimeters, what is the rate of change in the area of the region outside the square but inside the circle? Include units.

Answer Key:

1) Volume is increasing at a rate of $28\pi \text{ units}^3/\text{min}$.

$$\frac{dS}{dt} = 10\pi \text{ unit}^2/\text{min}$$

2) Surface area is decreasing at a rate of $\frac{1}{3} \text{ ft}^2/\text{min}$

3) a) $1 \text{ cm}/\text{min}$ b) $36 \text{ cm}^2/\text{min}$

4) 8 inches

5) a) $1 \text{ ft}/\text{min}$ b) $40\pi \text{ ft}^2/\text{min}$

6) a) $540 \text{ cm}^3/\text{sec}$ b) $180 \text{ cm}^2/\text{sec}$ c) 4 cm

7) a) $V = \frac{25\pi}{432} h^3$ b) $1.558 \text{ ft}/\text{min}$ c) $3\pi \text{ ft}/\text{min}$

8) a) $8\pi \text{ cm}^2$ b) $\frac{16}{5}\sqrt{2}\pi \text{ cm}^2/\text{sec}$ **or** $14.217 \text{ cm}^2/\text{sec}$

c) $8\sqrt{2}\pi - 16\sqrt{2}$ **or** $12.916 \text{ cm}^2/\text{sec}$