*Unit #1:* Integration Review
*Topic:* Riemann and Trapezoidal sums
*Objective: SWBAT find the area under a curve by using Riemann and Trapezoidal sums.*

# Warm Up #5:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 9.3 | 9.0 | 8.3 | 6.5 | 2.3 | −7.6 | −10.5 |

The table above gives the values of a function obtained from an experiment. Using three equal subintervals estimate the area of the region bounded by the graph of the function and the $x - axis$ from $x = 0$ to $x = 6$.

(a) left endpoints (LRAM)

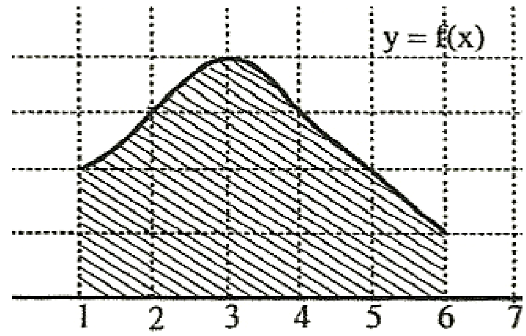
(b) right endpoints (RRAM)

(c) midpoints (MRAM)

(d) trapezoidal rule (TRAP)

***Problem Set #5:*** *Read each of the following carefully and show all work.*

1) Estimate the area bounded by the curve and the $x$-axis on $[1, 6]$ using the 5 equal subintervals by finding:




a) a left Riemann sum

b) a right Riemann sum

c) a midpoint Riemann sum

2) Approximate the area of the region bounded by the graph of $y = sinx$ and the $x - axis$ from $x = 0$ to $x = \pi$ using 3 equal subintervals using

(a) left endpoints (LRAM)

(b) right endpoints (RRAM)

(c) midpoints (MRAM)

(d) trapezoidal rule (TRAP)

3)

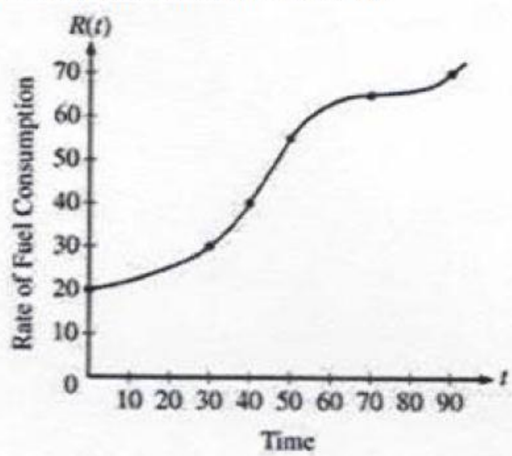
| $t$ (seconds) | 0 | 8 | 20 | 25 | 32 | 40 |
|---|---|---|---|---|---|---|
| $v(t)$ (meters per second) | 3 | 5 | $-10$ | $-8$ | $-4$ | 7 |

The velocity of a particle moving along the $x$-axis is modeled by a differentiable function $v$, where the position $x$ is measured in meters, and time $t$ is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

4) The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by the twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of R(t), for the time interval $0 \leq t \leq 90$ minutes, are shown below.




| $t$ (minutes) | $R(t)$ (gallons per minute) |
|---|---|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

Approximate the value for the area of the region bounded by the function above and the $x-axis$ using a left Riemann sum with five subintervals indicated by the data in the table. Is this numerical approximation less than or greater than the actual value of the area? Explain your reasoning.

# Warm Up #6:

2014 BC Exam #4

Train $A$ runs back and forth on an east-west section of railroad track. Train $A$'s velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time $t$ is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

| $t$ (minutes) | 0 | 2 | 5 | 8 | 12 |
|---|---|---|---|---|---|
| $v_A(t)$ (meters/minute) | 0 | 100 | 40 | −120 | −150 |

(a) Find the average acceleration of train $A$ over the interval $2 \le t \le 8$.

(b) Do the data in the table support the conclusion that train $A$'s velocity is $-100$ meters per minute at some time $t$ with $5 < t < 8$? Give a reason for your answer.

(c) At time $t = 2$, train $A$'s position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train $A$, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

(d) A second train, train $B$, travels north from the Origin Station. At time $t$ the velocity of train $B$ is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train $A$ and train $B$ is changing at time $t = 2$.

***Problem Set #6:*** *Read each of the following carefully and show all work.*

5) During a recent snowfall, several students monitored the accumulation of snow on the flat roof of their school. The table below records the data they collected for the 12-hour period of the snowfall.

| Number of Hours | Rate of Snowfall (in./hr) |
|---|---|
| 0 | 0 |
| 3 | 2.1 |
| 5 | 2.4 |
| 9 | 2.2 |
| 12 | 1.6 |

a) Use a right-hand Riemann sum with four subintervals to approximate the total depth of snow in the 12-hour period.

b) Using the right-hand sum approximation, estimate the average rate of snowfall in the 12-hour period.

6) The table below gives values of a continuous function. Use a midpoint Riemann sum with three equal subintervals to estimate the average value of $f$ on [20, 50].

| $x$ | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 42 | 38 | 31 | 29 | 35 | 48 | 60 |

7) The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function $W$ of time $t$. The table above shows the water temperature as recorded every 3 days over a 15-day period.

| $t$ (days) | $W(t)$ (°C) |
|---|---|
| 0 | 20 |
| 3 | 31 |
| 6 | 28 |
| 9 | 24 |
| 12 | 22 |
| 15 | 21 |

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

8)

| Distance $x$ (cm) | 0 | 1 | 5 | 6 | 8 |
|---|---|---|---|---|---|
| Temperature $T(x)$ (°C) | 100 | 93 | 70 | 62 | 55 |

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire $x$ cm from the heated end. The function $T$ is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

9)

| $t$ (minutes) | 0 | 2 | 5 | 9 | 10 |
|---|---|---|---|---|---|
| $H(t)$ (degrees Celsius) | 66 | 60 | 52 | 44 | 43 |

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function $H$ for $0 \le t \le 10$, where time $t$ is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time $t$ are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.