

Unit #7: Differentiation

Topic: Chain Rule

Objective: SWBAT find the derivatives of complex functions by using the chain rule.

Warm Up #5:

Find the derivative of each of the following functions:

$$1) f(x) = \frac{1}{2x - x^2}$$

$$2) g(x) = (3x^2 + 1)^2$$

Would you like to see another rule??

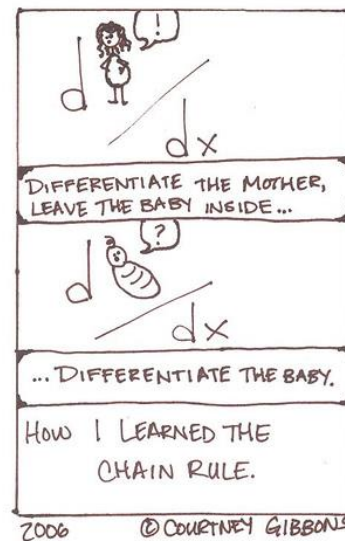


The **chain rule** can be used to find the derivative of a complex function.

The rule involves taking the derivative of the outside and multiplying it by the derivative of the inside:

Rule 6 The Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$



Example #1: Find $\frac{dy}{dx}$ if $f(x) = \sqrt{4x^2 - 3}$

Example #2: Find $g'(x)$ if $g(x) = -3\sin 5x$

Example #3: Find $\frac{d}{dx} 2(-x^3 + 5x^2)^4$

Problem Set #5: Find the derivative for each of the following.

1) $f(x) = (2x^5 - 7x)^3$

2) $h(x) = -3\cos x^5$

3) $g(x) = -\frac{5}{\sqrt{x^2 - 4x + 1}}$

4) $f(x) = (7 - 4x^2)^{2/3}$

5) $g(x) = \cot 4x - 3\cos 2x$

6) $f(x) = \sqrt[3]{x^2 - 5}$

$$7) h(x) = \sqrt[5]{(4x^3 - x)^2}$$

$$8) m(x) = \frac{1}{(2x + 1)^3}$$

$$9) f(x) = (1 - 2\cos x)^5$$

$$10) g(x) = \sec(3x - 1)$$

$$11) g(x) = -(3x^2 - 4x + 1)^4$$

$$12) g(x) = \frac{1}{2}\sqrt{5x^3}$$

$$13) m(x) = \cos(x^2 - x)$$

$$14) h(x) = \sqrt[4]{2x^7 - 3x}$$

$$15) f(x) = 8(1 - 5x^2)^{3/4}$$

$$16) g(x) = -2\csc 4x + 6\sqrt{7x}$$

Warm Up #6:

If $y = \frac{1}{2}x^3(2x^2 - 5x + 2)$, find

a) $\frac{dy}{dx}$

b) $\frac{d^2y}{dx^2}$

c) $\frac{d^3y}{dx^3}$

What if we had to do a product/quotient rule and a chain rule???



Example #4: Find $\frac{dy}{dx}$ if $y = x(2x^2 - 3)^4$.

Example #5: Find $g'(x)$ if $g(x) = \frac{x}{\sqrt{x^2-1}}$.

Problem Set #6: Find the derivative for each of the following.

$$17) f(x) = \sqrt{x} (1 - 2x)^2$$

$$18) g(x) = \frac{-3x}{(x^3 - x^2 + 3)^3}$$

$$19) h(x) = x \sin(4x^3)$$

$$20) p(x) = (x^2 + 2x - 6)^2 (1 - x^3)^2$$

$$21) f(x) = \frac{3}{\sqrt{a^3 + 2}}$$

$$22) g(x) = x^3 (5x - 1)^4$$

$$23) h(x) = \frac{\cot(5x)}{2x}$$

$$24) m(x) = \frac{1}{(x^5 - x + 3)^6}$$

$$25) f(x) = 2x \sec(3x^2)$$

$$26) h(x) = \frac{4}{\sqrt{6x + 17}}$$

$$27) g(x) = (6x^2 + 5)^3(x^3 - 7)^4$$

$$28) f(x) = \frac{x}{\sqrt[3]{(1 - 2\cos x)^2}}$$

$$29) f(x) = \frac{-7}{(2x+7)^2}$$

$$30) g(x) = (2x - 1)(\sin x)^3$$

$$31) f(x) = \frac{1}{\sqrt[3]{3x^3 - 4x + 2}}$$

$$32) h(x) = -8x(4 - 9x)^{3/2}$$

Warm Up #7:

If $f(x) = 7\sin^2x$, find $f''\left(\frac{\pi}{2}\right)$.

Ok, so now what if we needed to use the chain rule more than once???

Example #6: Find $\frac{d}{dx}[\cos^3 5x]$.



Example #7: Find $\frac{dy}{dx}$ if $y = (2\sqrt{3x-1} + 5)^4$.

Problem Set #7: Find the derivative for each of the following.

33) $h(x) = 4\sin^5(x^2)$

34) $y = (1 + \cos^2 7x)^3$

35) $g(x) = \tan(5 - \sin 2\pi x)$

36) $f(x) = 3\sqrt{\cot 5x}$

37) $y = 9\tan^2\left(\frac{x}{3}\right)$

38) $g(x) = \csc^3(2x)$

39) $f(x) = (2 - \sqrt{3x - 5})^4$

40) $y = \sqrt{3\cos^2 x + 2}$