Unit #5: Improper Integrals *Topic:* Double-Sided Improper Integrals *Objective: SWBAT integrate improper integrals with infinite discontinuities by using limits*

Warm Up #5:

Identify any points of discontinuity for each of the following and identify the type of discontinuity:

a)
$$f(x) = \frac{3x}{2x-6}$$
 b) $f(x) = \frac{x-1}{x^2-5}$

c)
$$f(x) = \frac{x^2 - 4}{x - 2}$$
 d) $f(x) = \frac{x^2 - 9}{x^2 - x - 12}$

Evaluating Improper Integrals Case 2

Case 2: Integrands with Infinite Discontinuities

Another type of improper integral occurs when the integrand has a vertical asymptote at a limit of integration or at some point between the limits of integration. In order to evaluate these integrals, we use the following methods.

1) If f(x) is continuous on (a, b] and discontinuous at a, then

$$\int_{a}^{b} f(x)dx = \lim_{c \to a^{+}} \int_{c}^{b} f(x)dx$$

2) If f(x) is continuous on [a, b] and discontinuous at b, then

$$\int_{a}^{b} f(x)dx = \lim_{c \to b^{-}} \int_{a}^{c} f(x)dx$$

3) If f(x) is discontinuous at c, where a < c < b, and continuous on $[a, c) \cup (c, b]$, then

$$\int_{a}^{b} f(x)dx = \int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx$$

Example #1: Find the value of $\int_0^1 \frac{dx}{\sqrt{x}}$

Example #2:
Find the value of
$$\int_{1}^{2} \frac{1}{x-2} dx$$

Example #3: Find the value of $\int_0^2 \frac{dt}{(1-t)^{1/3}}$ *Problem Set #5: Find the value of each of the following integrals and determine whether they are converging or diverging.*

$1. \int_{2}^{3} \frac{dt}{(3-t)^2}$	2. $\int_{-2}^{2} \frac{dx}{x^2}$
$3. \int_0^1 \frac{1}{\sqrt[3]{x}} dx$	4. $\int_0^1 \frac{1}{x^3} dx$
5. $\int_0^2 \frac{dx}{(x-1)^{2/3}}$	$6. \int_0^4 \frac{1}{\sqrt{4-x}} dx$

7. $\int_0^1 \frac{x+1}{\sqrt{x^2+2x}} dx$	8. $\int_0^2 \frac{dx}{1-x^2}$
9. $\int_0^{\pi/2} \frac{\sin x}{\sqrt{1 - \cos x}} dx$	10. $\int_0^{\ln 2} y^{-2} e^{1/y} dy$
11. $\int_0^{\pi/2} tanx dx$	12. $\int_0^1 lnx dx$
Answers:	'
1. D 2. D 3. 3/2 4. D 5. 6 6. 4	7. $\sqrt{3}$ 8. D 9. 2 10. D 11. D 121