

Unit #5: Improper Integrals

Topic: Double-Sided Improper Integrals

Objective: SWBAT integrate improper integrals with infinite discontinuities by using limits

Warm Up #5:

Identify any points of discontinuity for each of the following and identify the type of discontinuity:

a) $f(x) = \frac{3x}{2x - 6}$

b) $f(x) = \frac{x - 1}{x^2 - 5}$

c) $f(x) = \frac{x^2 - 4}{x - 2}$

d) $f(x) = \frac{x^2 - 9}{x^2 - x - 12}$

Evaluating Improper Integrals Case 2Case 2: Integrands with Infinite Discontinuities

Another type of improper integral occurs when the integrand has a vertical asymptote at a limit of integration or at some point between the limits of integration. In order to evaluate these integrals, we use the following methods.

1) If $f(x)$ is continuous on $(a, b]$ and discontinuous at a , then

$$\int_a^b f(x) dx = \lim_{c \rightarrow a^+} \int_c^b f(x) dx$$

2) If $f(x)$ is continuous on $[a, b)$ and discontinuous at b , then

$$\int_a^b f(x) dx = \lim_{c \rightarrow b^-} \int_a^c f(x) dx$$

3) If $f(x)$ is discontinuous at c , where $a < c < b$, and continuous on $[a, c) \cup (c, b]$, then

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

Example #1:

Find the value of $\int_0^1 \frac{dx}{\sqrt{x}}$

Example #2:

Find the value of $\int_1^2 \frac{1}{x-2} dx$

Example #3:

Find the value of $\int_0^2 \frac{dt}{(1-t)^{1/3}}$

Problem Set #5: Find the value of each of the following integrals and determine whether they are converging or diverging.

1. $\int_2^3 \frac{dt}{(3-t)^2}$

2. $\int_{-2}^2 \frac{dx}{x^2}$

3. $\int_0^1 \frac{1}{\sqrt[3]{x}} dx$

4. $\int_0^1 \frac{1}{x^3} dx$

5. $\int_0^2 \frac{dx}{(x-1)^{2/3}}$

6. $\int_0^4 \frac{1}{\sqrt{4-x}} dx$

7. $\int_0^1 \frac{x+1}{\sqrt{x^2+2x}} dx$

8. $\int_0^2 \frac{dx}{1-x^2}$

9. $\int_0^{\pi/2} \frac{\sin x}{\sqrt{1-\cos x}} dx$

10. $\int_0^{\ln 2} y^{-2} e^{1/y} dy$

11. $\int_0^{\pi/2} \tan x dx$

12. $\int_0^1 \ln x dx$

Answers:

1. D 2. D 3. 3/2 4. D 5. 6 6. 4 7.
- $\sqrt{3}$
8. D 9. 2 10. D 11. D 12. -1