Unit 5: Limits
Topic: Infinite Limits
Objective: SWBAT use infinite limits to find vertical asymptotes.

## Warm Up \#5:

a) Graph (without a calculator) $f(x)=\frac{1}{x}$
b) Find $\lim _{x \rightarrow 0^{-}} f(x)$
c) Find $\lim _{x \rightarrow 0^{+}} f(x)$


## Infinite Limits

Infinity is a very special idea. It is not a concrete number, but a really long never ending journey.

In general, we write symbolically
$\lim _{x \rightarrow c} \boldsymbol{f}(\boldsymbol{x})=\infty$ to indicate that the values of $f(x)$ tend to become larger and larger or
$\lim _{x \rightarrow \boldsymbol{c}} \boldsymbol{f}(\boldsymbol{x})=-\infty$ to indicate that the values of $f(x)$ tend to become smaller and smaller

This does not mean that we are regarding $\infty$ as a number. Nor does it mean that the limit exists. It simply expresses the particular way in which the limit does not exist.

Knowing where a particular function has vertical asymptotes will allow you to quickly evaluate limits at these $x$-values.

Example \#1: For the function $R$ whose graph is shown, state the following.
(a) $\lim _{x \rightarrow 2} R(x)$
(b) $\lim _{x \rightarrow 5} R(x)$
(c) $\lim _{x \rightarrow-3^{-}} R(x)$
(d) $\lim _{x \rightarrow-3^{+}} R(x)$
(e) The equations of the vertical asymptotes.

|  |  |  |  |  |  | $y^{\prime}$ |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
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Example \#2: Find each of the following limits without a calculator.
a) $\lim _{x \rightarrow 1} \frac{(2-x)}{(x-1)^{2}}$
b) $\lim _{x \rightarrow-3^{+}} \frac{-2}{(x+3)}$
c) $\lim _{x \rightarrow 3^{+}}\left(x-3-\frac{1}{x-3}\right)$

## Practice Problems:

1) For the function $f$ whose graph is shown, state the following.
(a) $\lim _{x \rightarrow-7} f(x)$
(b) $\lim _{x \rightarrow-3} f(x)$
(c) $\lim _{x \rightarrow 0} f(x)$
(d) $\lim _{x \rightarrow 6^{-}} f(x)$
(e) $\lim _{x \rightarrow 6^{+}} f(x)$
(f) The equations of the vertical asymptotes.

2) Find the limit for each of the following:

| a) $\lim _{x \rightarrow 1^{+}} \frac{1}{x-1}$ | b) $\lim _{x \rightarrow-3} \frac{1}{(x+3)^{2}}$ |
| :--- | :--- |
| c) $\lim _{x \rightarrow-3^{+}} \frac{x+2}{x+3}$ | d) $\lim _{x \rightarrow 2^{-}} \frac{x^{2}-2 x}{x^{2}-4 x+4}$ |


| e) $\lim _{x \rightarrow 5^{-}} \frac{1}{5-x}$ | f) $\lim _{x \rightarrow 5^{-}} \frac{1}{(5-x)^{2}}$ |
| :---: | :---: |
| g) $\lim _{x \rightarrow 2} \frac{-1}{(x-2)^{2}}$ | h) $\lim _{x \rightarrow \frac{\pi}{2}} \tan x$ |
| i) $\lim _{x \rightarrow 2^{+}} \frac{1}{x^{2}-4}$ | j) $\lim _{x \rightarrow-2^{+}} \frac{1}{x^{2}-4}$ |
| k) $\lim _{x \rightarrow-2^{-}} \frac{x^{2}-1}{2 x+4}$ | 1) $\lim _{x \rightarrow 1^{+}} \frac{x^{2}-2 x}{x-1}$ |
| m) $\lim _{x \rightarrow 2^{-}} \frac{x}{x-2}$ | n) $\lim _{x \rightarrow-1^{-}}\left(1-x-\frac{1}{x+1}\right)$ |

