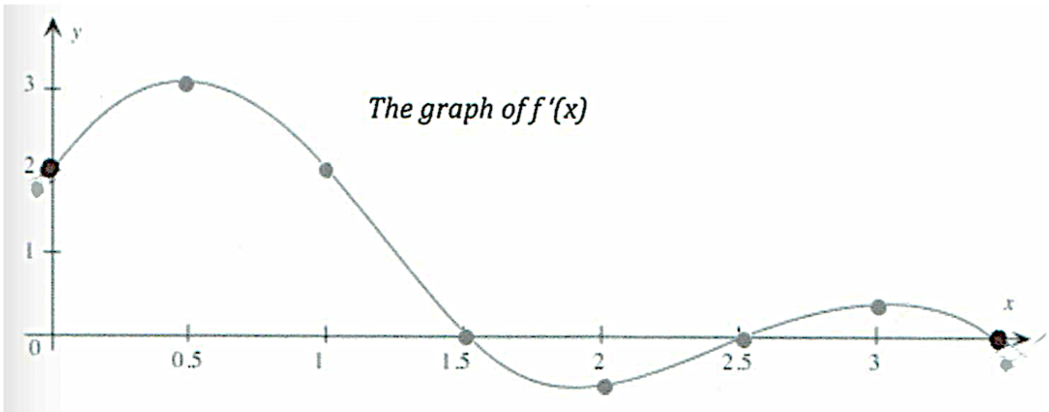
Unit #10: Applications of Differentiation
Topic: Second Derivative Test
Objective: SWBAT describe the concavity of a function by using the second derivative.

Warm Up #5:



Let f be a function defined for all real numbers. The graph of f', the derivative of f, is shown above.

(a) Find all values of x at which f has a relative maximum. Justify your answer.

(b) Find all values of x at which f has a relative minimum. Justify your answer.

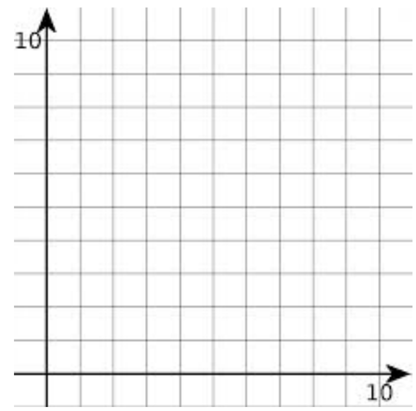
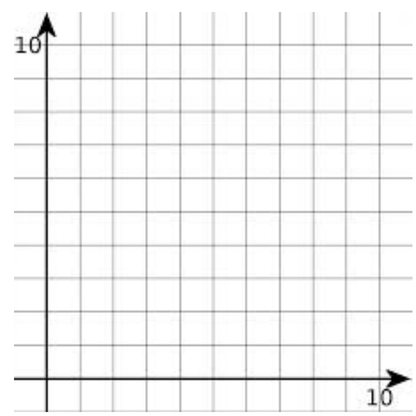
(c) Find all intervals on which the graph of f is increasing.

(d) Find all intervals on which the graph of f is decreasing.

If f' tells us how the $y - values$ of a graph of f are changing, then the derivative of f', or f'', tells us how the **slopes** of f are changing.

Concavity and the 2nd Derivative Test

For a continuous function $f(x)$ on an interval,

What does it sound like??	*What does it look like??*
If ______________________, then f is __________________________________. AND If __________________ and ___________________ , then $(c, f(c))$ is a ________________________________.	
If ______________________, then f is __________________________________. AND If __________________ and ___________________ , then $(c, f(c))$ is a ________________________________.	
An _____________________, in the domain of f, where the graph changes concavity is called an ____ and the point $(c, f(c))$ is a ____. A graph can also change its concavity at a _________________________________ of $f''(x)$ as long as the $x - value$ is in the domain of the function.	

Example #1: Given $f(x) = \frac{x^2}{x^2+3}$, find (i) where f is increasing/decreasing, (ii) any relative minimum/maximum points, and (iii) the interval of concavity and inflection points of f.

Example #2: Find the intervals of concavity and inflection points of $g(x) = \frac{1}{x-3}$.

Directions: For each of the following find (i) where f is increasing/decreasing, (ii) any relative minimum/maximum points, and (iii) the intervals of concavity and inflection points of f.

1) $f(x) = x^3 - 2x^2 - 2$

2) $f(x) = xe^x$

Directions: For each of the following find the intervals of concavity and inflection points of f.

3) $f(x) = 4x^3 + 21x^2 + 36x - 20$

4) $f(x) = -\frac{3}{16}(x-1)^{4/3} - \frac{3}{2}(x-1)^{1/3} + 2$

ANSWER KEY

1) (i) increasing $(-\infty, 0)(\frac{4}{3}, \infty)$ decreasing $(0, \frac{4}{3})$ (ii) rel. min $x = 4/3$ and rel max $x = 0$ (iii) concave up $(\frac{2}{3}, \infty)$ concave down $(-\infty, \frac{2}{3})$ and inflection pt. at $x = 2/3$
2) (i) increasing $(-1, \infty)$ decreasing $(-\infty, -1)$ (ii) rel. min $x = -1$ (iii) concave up $(-2, \infty)$ concave down $(-\infty, -2)$ and inflection pt. at $x = -2$
3) concave up $(-\frac{7}{4}, \infty)$ concave down $(-\infty, -\frac{7}{4})$ and inflection pt. at $x = -7/4$
4) concave up $(1,5)$ concave down $(-\infty, 1)(5, \infty)$ and inflection pt. at $x = 1$ *and* 5