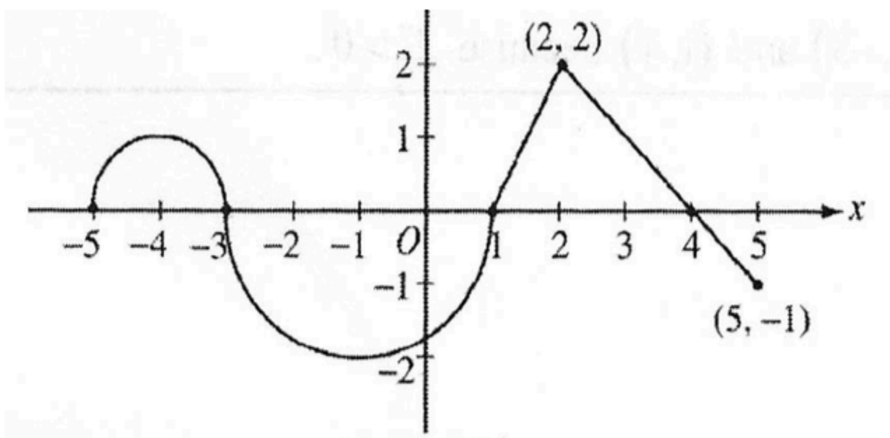
Unit #10: Applications of Differentiation
Topic: Connecting the Graphs of $f, f', and\ f''$.
Objective: SWBAT use their knowledge of the first and second derivative to solve various AP exam practice problems.

Warm Up #6:

Let f be a function defined on the closed interval $-5 \leq x \leq 5$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown below.



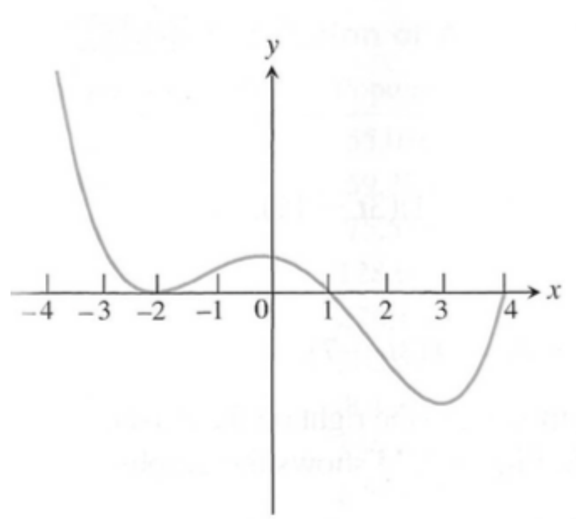
Graph of f'

(a) For $-5 < x < 5$, find all values of x at which f has a relative maximum. Justify your answer.
(b) For $-5 < x < 5$, find all values of x at which f has a point of inflection. Justify your answer.
(c) Find all intervals on which the graph of f is concave up. Justify your answer.
(d) Find all the intervals on which the graph of f has a positive slope. Justify your answer.

All of our work with the first and second derivative will now help us to graph unfamiliar functions without a calculator and/or the equation of the original function.

Example #1:

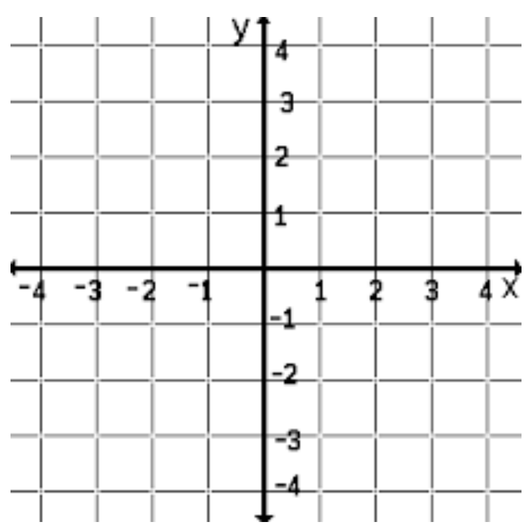
The graph of a derivative of a function f on the interval [-4,4] is shown below.



(a) On what intervals is f increasing?

(b) On what intervals is the graph of f concave up?

(c) At which x −coordinate does f have local extrema?



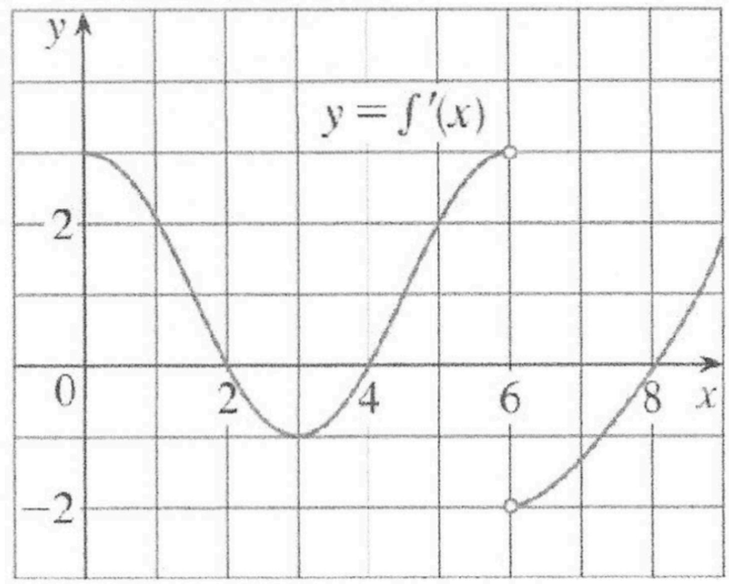
(d) What are the x −coordinates of all inflection points of the graph of f?

(e) Sketch a possible graph of f on the interval [-4,4].

Example #2:

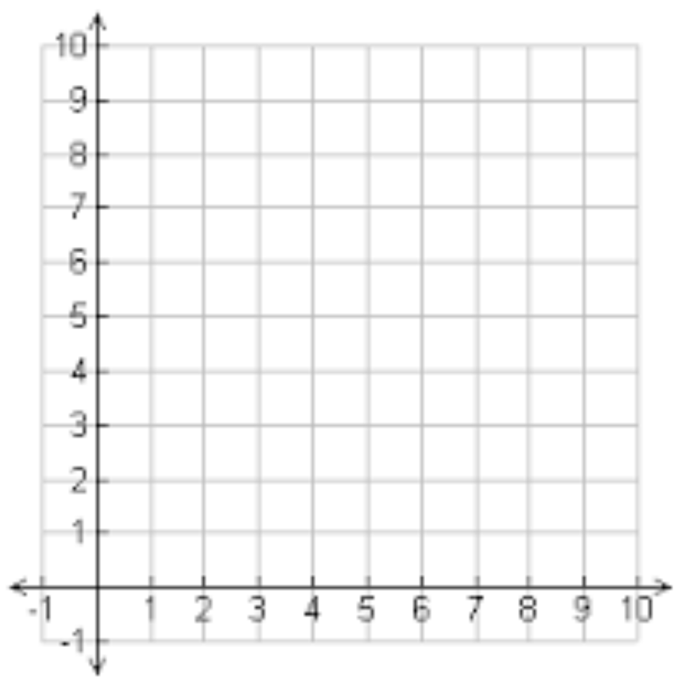
The graph of a derivative f' of a continuous function on $[0,9]$ is shown below.

(a) On what open intervals is f increasing or decreasing? Justify your answer.



(b) At what values of x does f have a local maximum or minimum? Justify your answer.

(c) On what intervals is f concave up or concave down? Justify your answer.



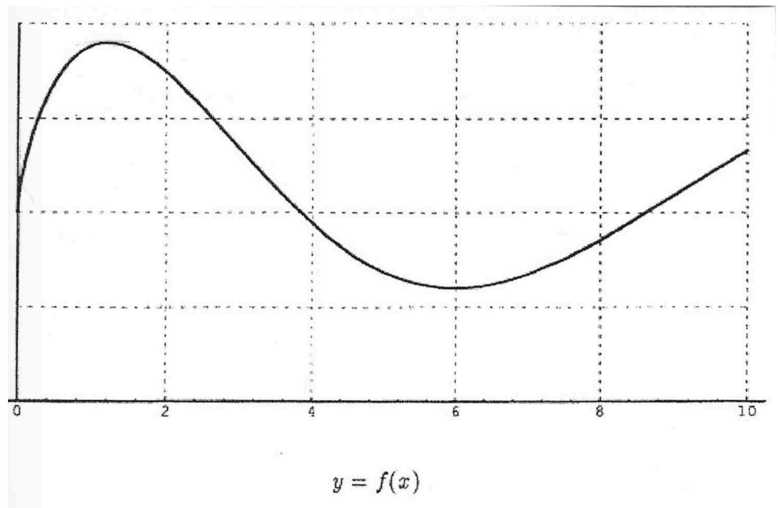
(d) State the x −coordinate(s) of the point(s) of inflection.

(e) Assuming that $f(0) = 0$, sketch the graph of f.

Example #3:

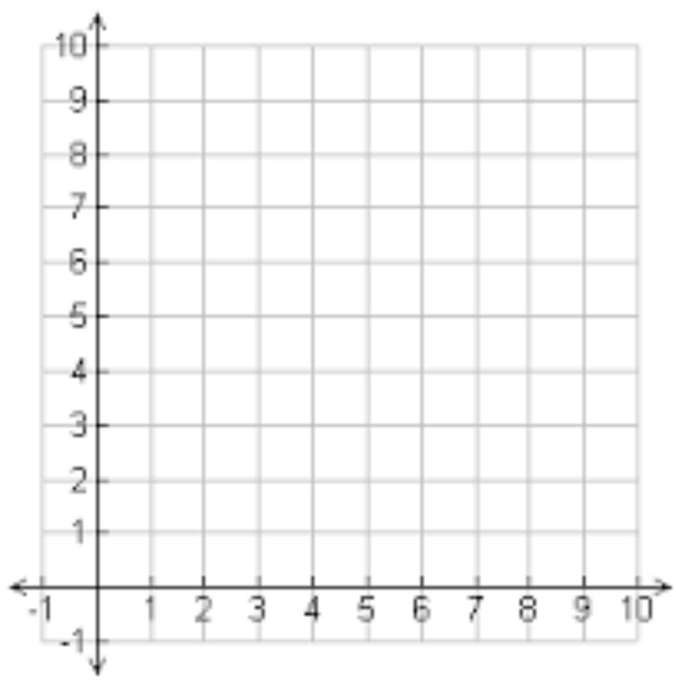
The graph of a continuous function, f, on $[0,10]$ is shown below.

(a) On what open intervals is f increasing or decreasing? Justify your answer.



(b) At what values of x does f have a local maximum or minimum? Justify your answer.

(c) On what intervals is f concave up or concave down? Justify your answer.

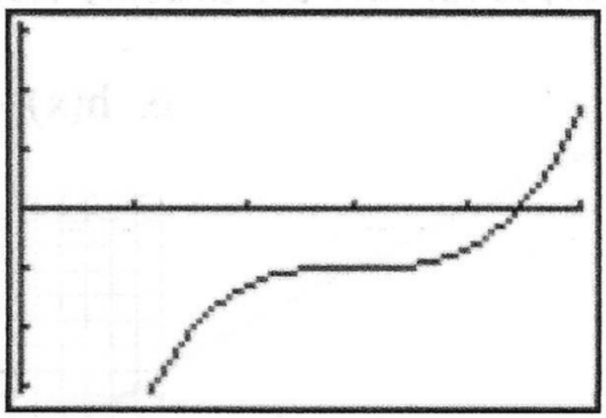
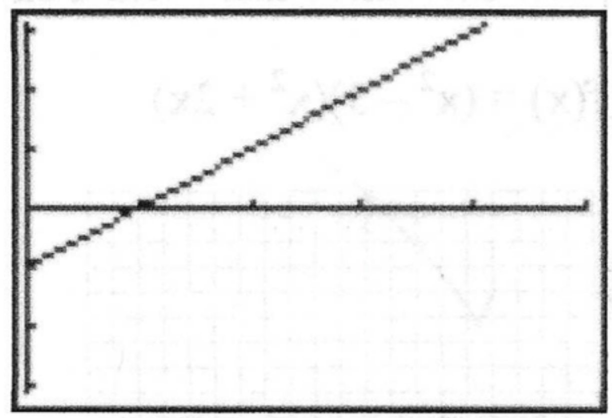
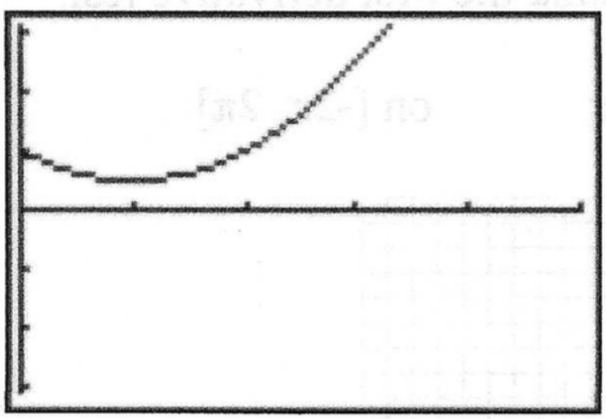
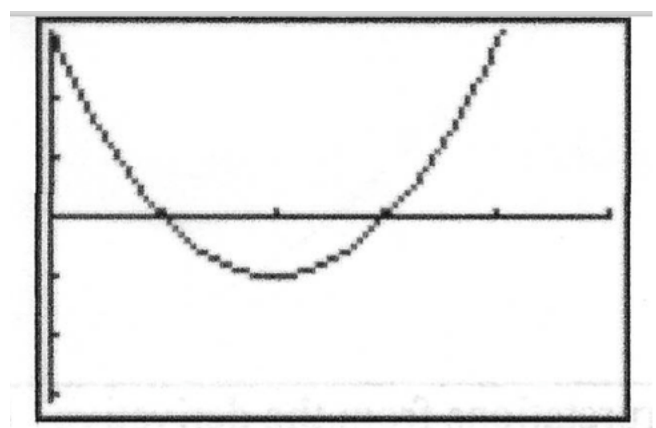
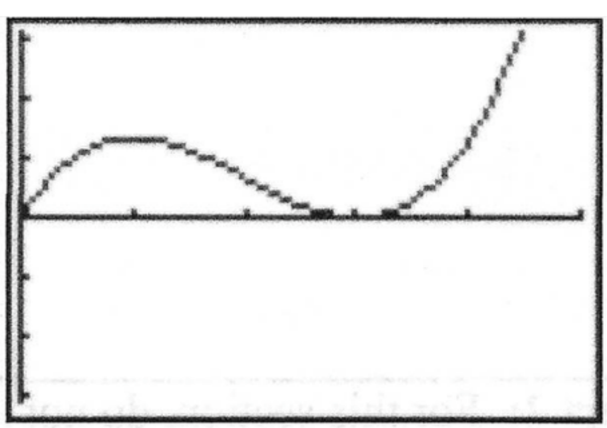
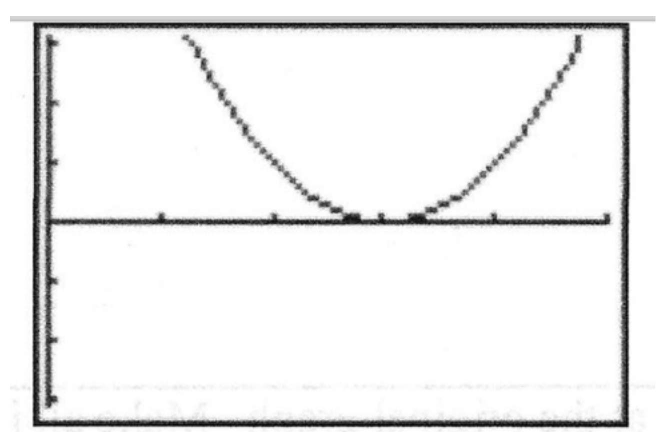


(d) State the x −coordinate(s) of the point(s) of inflection.

(e) Sketch the graph of $f\,'$.

Warm Up #7:

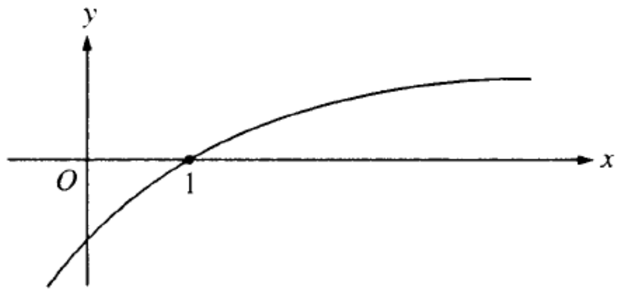
Match each of the following functions with the graph of their derivatives.

f	***f′***
1) 	a) 
2) 	b) 
3) 	c) 

For each of the following use your knowledge of the relationships between $f, f', and\ f''$ to make conclusions based on certain given information.

Think VERY CAREFULLY about each question!!

1)



The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

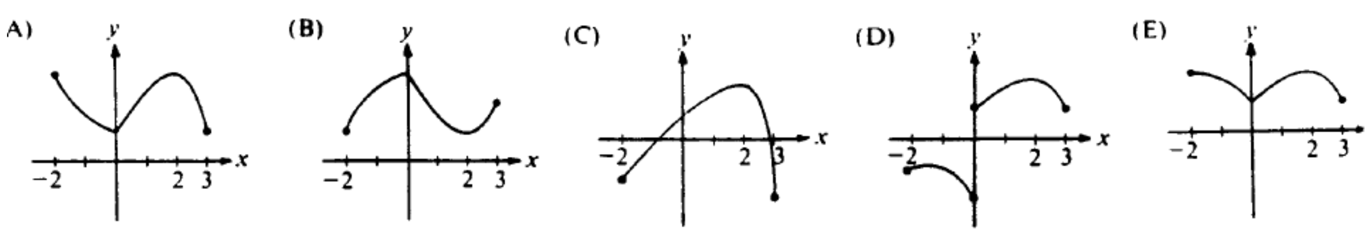
(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

2) Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2)=0$, and $f''(x)<0$ for all x except $x=0$. Which of the following could be the graph of f?



3)

x	0	1	2	3
$f''(x)$	5	0	–7	4

The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

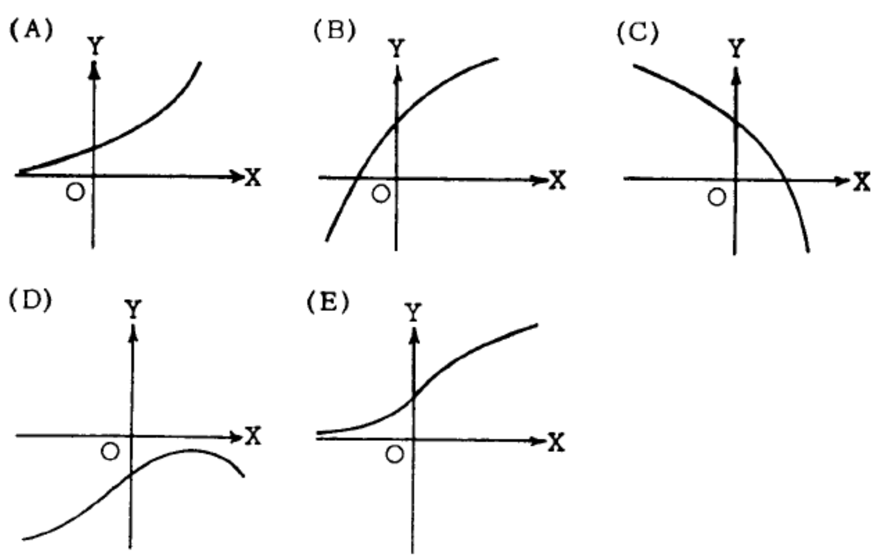
(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

4) If y is a function x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



5) A polynomial $p(x)$ has a relative maximum at $(-2,4)$, a relative minimum at $(1,1)$, a relative maximum at $(5,7)$ and no other critical points. How many zeros does $p(x)$ have?

(A) One (B) Two (C) Three (D) Four (E) Five

6)

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

The function f is continuous and differentiable on the closed interval $[0, 4]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

(A) The minimum value of f on $[0, 4]$ is 2.

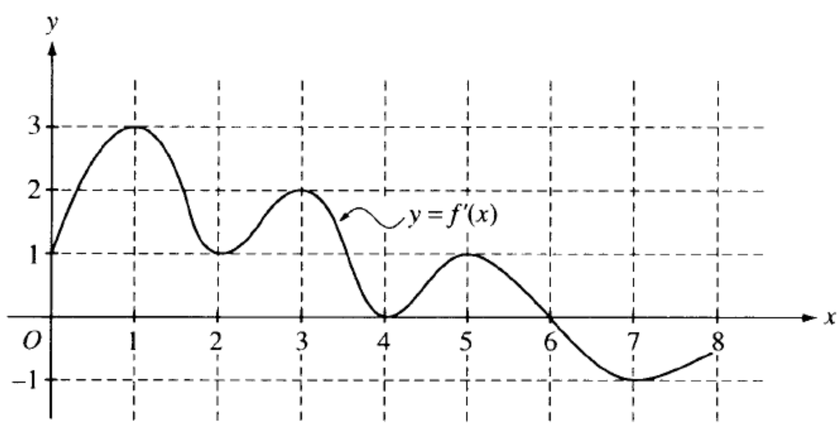
(B) The maximum value of f on $[0, 4]$ is 4.

(C) $f(x) > 0$ for $0 < x < 4$

(D) $f'(x) < 0$ for $2 < x < 4$

(E) There exists c, with $0 < c < 4$, for which $f'(c) = 0$.

7)



The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above.

The point $(3,5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3,5)$ is

(A) $y = 2$

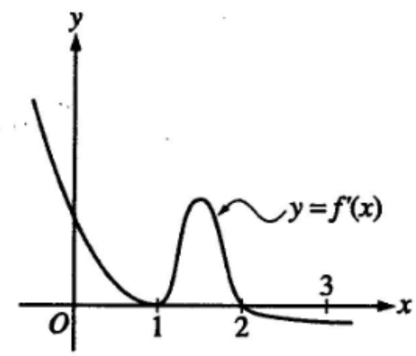
(B) $y = 5$

(C) $y - 5 = 2(x - 3)$

(D) $y + 5 = 2(x - 3)$

(E) $y + 5 = 2(x + 3)$

8)



The graph of f', the derivative of the function f, is shown above. If $f(0) = 0$, which of the following must be true?

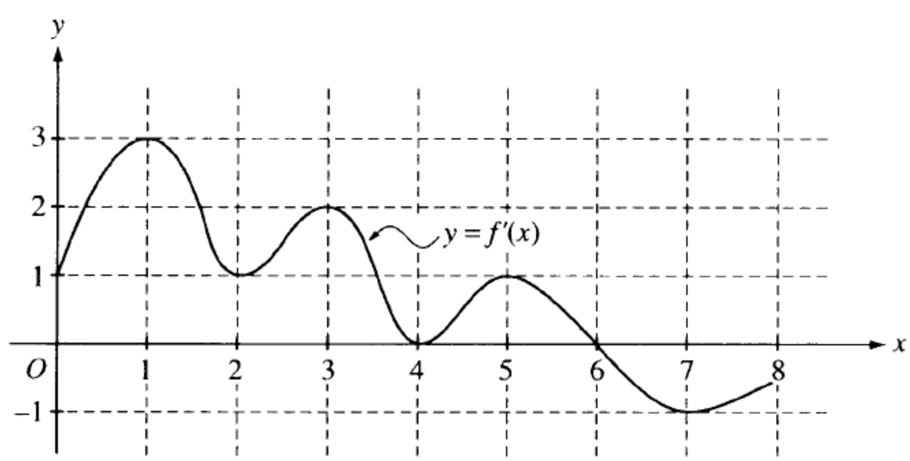
I. $f(0) > f(1)$

II. $f(2) > f(1)$

III. $f(1) > f(3)$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

9)

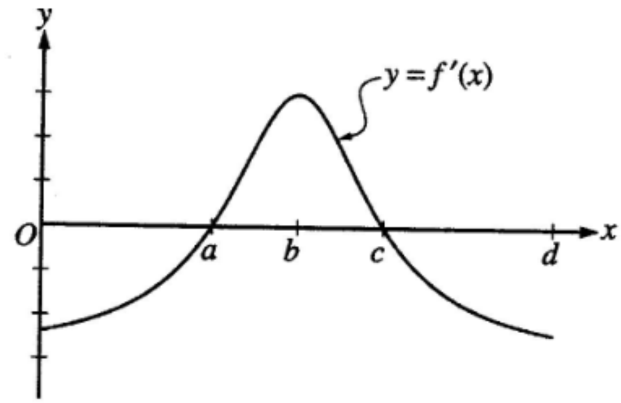


The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above.

How many points of inflection does the graph of f have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

10)



The graph of f', the derivative of a function f, is shown above. The domain of f is the open interval $0 < x < d$. Which of the following statements is true?

(A) f has a local minimum at $x = c$.

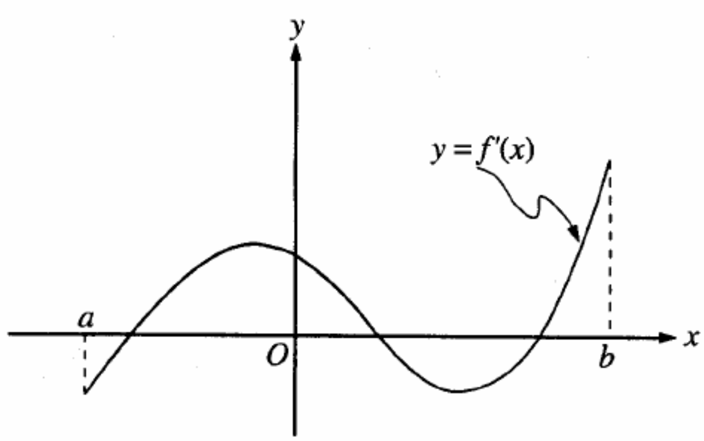
(B) f has a local maximum at $x = b$.

(C) The graph of f has a point of inflection at $(a, f(a))$.

(D) The graph of f has a point of inflection at $(b, f(b))$.

(E) The graph of f is concave up on the open interval (c, d).

11)



The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima