Unit #7: Sequence and Series Date: ________________

Topic: Tests for Convergence

Objective: SWBAT determine whether a series converges or diverges.

Warm Up #6:

What is the sum of each of the following?

a) $\sum_{n=0}^{\infty}\left(-\frac{3}{2}\right)^{-n}$

b) $\sum_{n=1}^{\infty}\left(\frac{3}{(2n-1)(2n+1)}\right)$

Comparison of Series

4. The Direct Comparison Test

First: $\sum a_n \geq 0$ and $\sum b_n \geq 0$

Second: You **MUST** be able to STATE/SHOW the inequality (relationship) that exists between a_n and b_n.

If $a_n \leq b_n$ and $\sum b_n$ converges, **then** $\sum a_n$ converges.

If $a_n \geq b_n$ and $\sum b_n$ diverges, **then** $\sum a_n$ diverges.

Example #4: Do each of the following series converge or diverge?

a) $\sum_{n=1}^{\infty} \frac{1}{3^n + 2}$

b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} - 1}$

Sometimes the inequalities needed above don't hold or are difficult to show, but you still recognize a similar series with which to compare it.

5. The Limit Comparison Test

Let $\sum a_n \geq 0$ and $\sum b_n \geq 0$ be two series where

$$\lim_{n \to \infty} \left| \frac{a_n}{b_n} \right| = L$$

If $L > 0$ **then** both series either converge or diverge. **If** $L = 0$ and $\sum b_n$ converges, **then** $\sum a_n$ converges. **If** $L = \infty$ and $\sum b_n$ diverges, **then** $\sum a_n$ diverges.

Example #5: Do each of the following series converge or diverge?

a) $\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$

b) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

Practice Problems: Determine whether each of the following series converges or diverges.

25) $\sum_{n=1}^{\infty} \frac{\sin n}{n^3 + 1}$	26) $\sum_{n=1}^{\infty} \frac{1}{1 + n^4}$

27) $\sum_{n=1}^{\infty} \frac{1}{2n+1}$	28) $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$
29) $\sum_{n=1}^{\infty} \frac{2+e^{-n}}{n}$	30) $\sum_{n=1}^{\infty} \frac{sin^2 n}{n^2}$
31) $\sum_{n=1}^{\infty} \frac{1}{n^{(1+1/n)}}$	32) $\sum_{n=1}^{\infty} \frac{cos n}{2^n}$

33) $\sum_{n=1}^{\infty} \frac{n^2-1}{5n^4+1}$	34) $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$
35) $\sum_{n=1}^{\infty} \frac{1+\sin n}{10^n}$	36) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3-2n^2+5}$
37) $\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^3-1}}$	38) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n}}$

Warm Up #7:

For what values of x does the series $1 + 2^x + 3^x + 4^x + \cdots + n^x + \cdots$ converge?

(A) no values of x (B) $x < -1$ (C) $x \geq -1$ (D) $x > -1$ (E) All values of x

6. The Alternating Series Test

If a_n is decreasing and the $\lim\limits_{n\to\infty} a_n = 0$, **then** $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ converges.

Example #6: Do the following series converge or diverge?

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

b) $\sum_{n=1}^{\infty}(-1)^{n-1}3^n$

Error Formula for an Alternating Series

If an alternating series converges, then its sum can be approximated by the sum of the first n terms of the series, and the **error** in the approximation is less than the $(n+1)^{st}$ term.

Example #7: If the first 5 terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ are used to approximate the sum, what is the error estimate on this approximation?

Absolute and Conditional Convergence

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges and the series of positive terms $\sum_{n=1}^{\infty} a_n$ converges, then the alternating series is said to be ***absolutely convergent.***

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges and the series of positive terms $\sum_{n=1}^{\infty} a_n$ diverges, then the alternating series is said to be ***conditionally convergent.***

Example #8: Are each of the following series absolutely or conditionally convergent?

a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

Practice Problems: Determine whether each of the following series converges or diverges. If it converges, does it converge conditionally or absolutely?

39) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{e^n}$	40) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^n}$
41) $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{n^2+9}$	42) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n+1}}$

43) $\sum_{n=1}^{\infty} \frac{n(-1)^{n+1}}{e^{n^2}}$	44) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$
45) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{2^n}$	46) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{sinn}{n^2}$

47) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1+\sqrt{n}}$	48) $\sum_{n=0}^{\infty} (-1)^{n+1} \left(\frac{3}{2}\right)^{n}$

Answer Key

25) Converges	26) Converges	27) Diverges
28) Converges	29) Diverges	30) Converges
31) Diverges	32) Converges	33) Converges
34) Diverges	35) Converges	36) Converges
37) Diverges	38) Converges	39) Absolutely Converges
40) Absolutely Converges	41) Diverges	42) Conditionally Converges
43) Absolutely Converges	44) Conditionally Converges	45) Diverges
46) Absolutely Converges	47) Conditionally Converges	48) Diverges