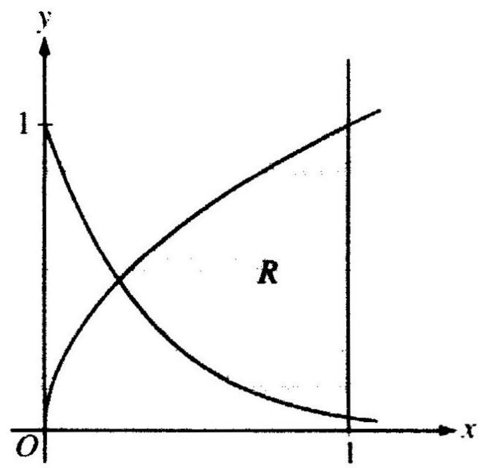
Unit #4: Area and Volume
Topic: Finding the Volume of Solids of Revolution
Objective: SWBAT find the volume of a solid of revolution using the disk/washer methods.

Warm Up #6:

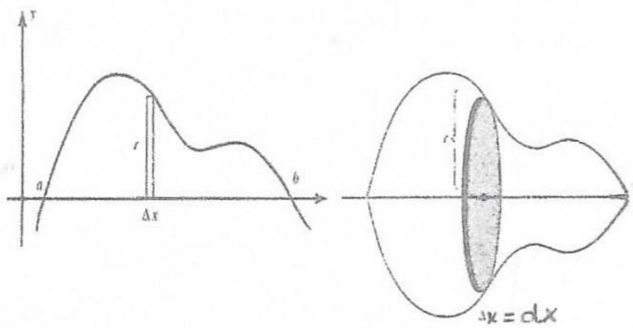


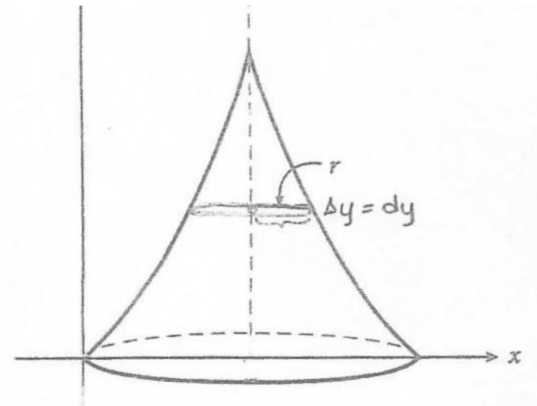
Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

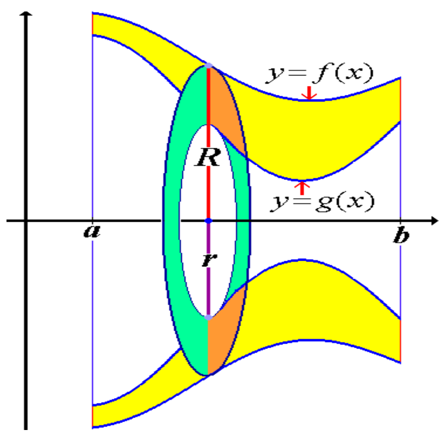
(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

A *solid of revolution* is obtained when a plane region is revolved about a fixed line, called the *axis of revolution*.







The disk/washer is a cylinder whose radii , *R and r*, represent the distances between the axis of revolution and the function. Thus,

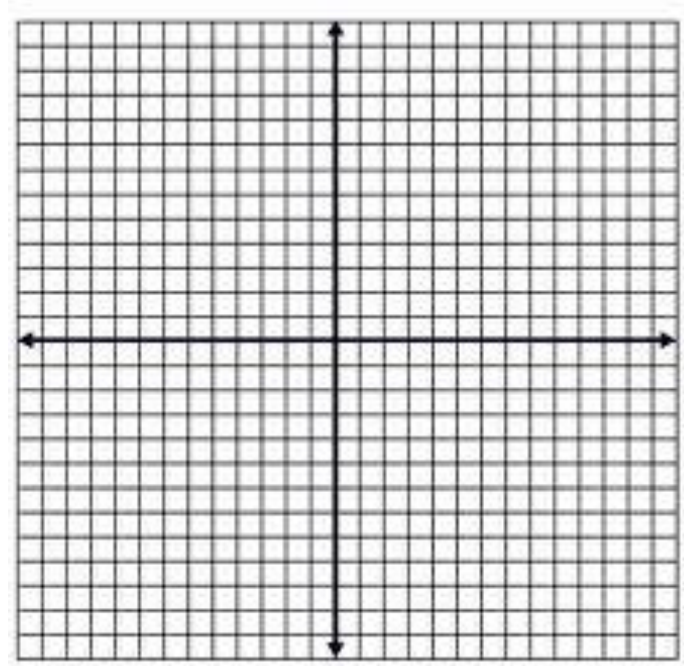
Horizontal Axis of Revolution	Vertical Axis of Revolution
$V = \pi \int_a^b (R^2 - r^2)dx$	$V = \pi \int_a^b (R^2 - r^2)dy$

Using these cylinders to find the volume for a given region is known as the **disk/washer method.**

Dealing with Disks:

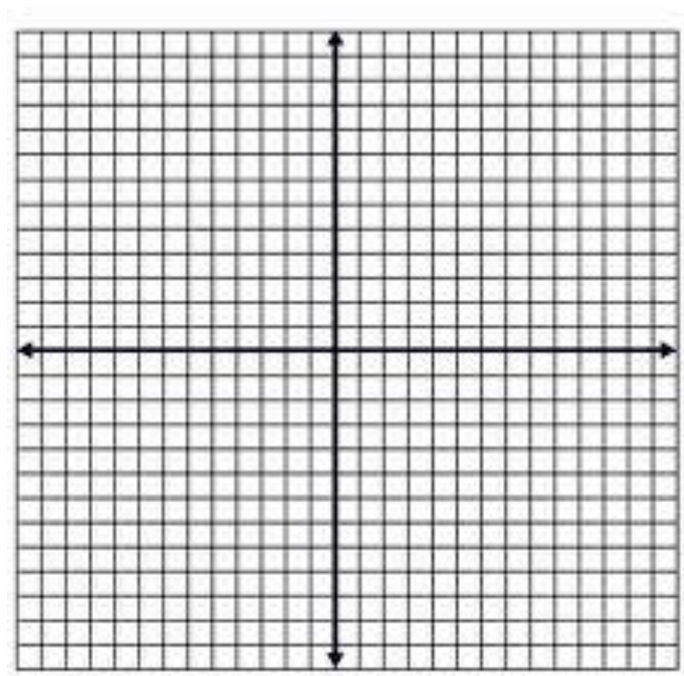
Example #1:

Using a calculator, find the volume of the solid generated by revolving about the line $y = -3$ the region bounded by the graph of $y = e^x$, the $y - axis$, the lines $x = ln2$ and $y = -3$.



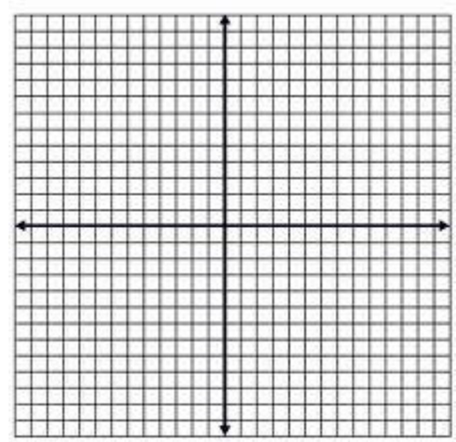
Example #2:

Find the volume of the solid generated when the region bounded by $y = x^2, x = 2,$ $and\ the\ x - axis$ is rotated about the line $x = 2$.

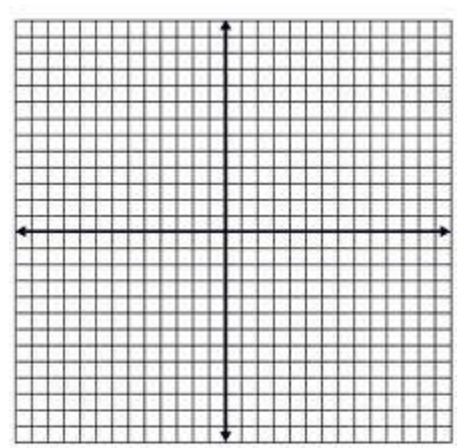


Problem Set #6:

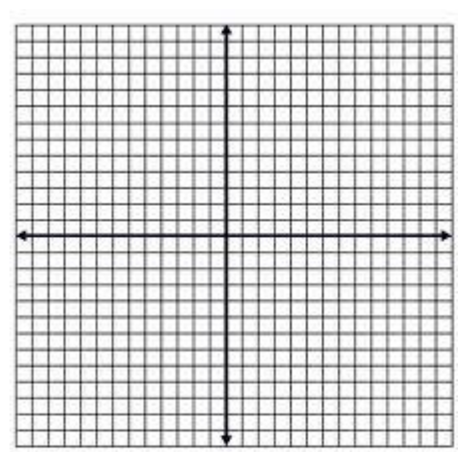
1. Find the volume of the solid generated by revolving about the x-axis the region bounded by the graph of $f(x) = \sqrt{x-1}$, the $x - axis$, and the line $x = 5$.



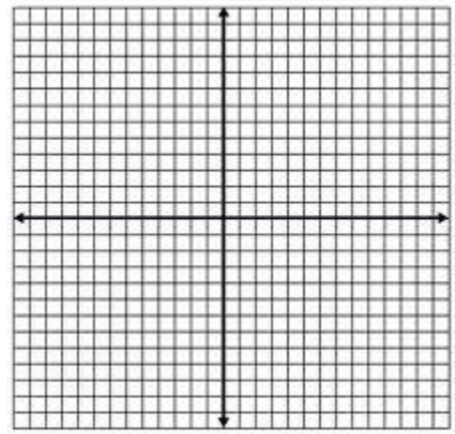
2. Find the volume of the solid generated by revolving about the $x - axis$ the region bounded by the graph of $y = \sqrt{cosx}$ where $0 \leq x \leq \frac{\pi}{2}$, the $x - axis$, and the $y - axis$.



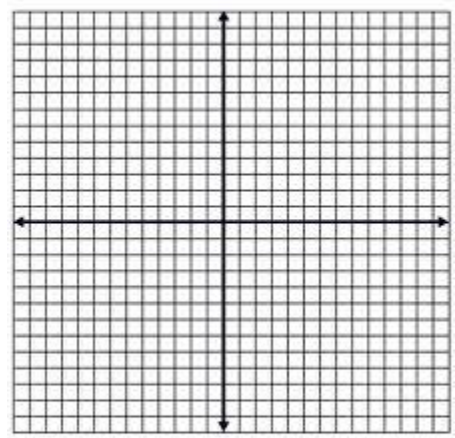
3. Find the volume of the solid generated by revolving about the $y - axis$ the region in the first quadrant bounded by the graph of $y = x^2$, the $y - axis$, and the line $y = 6$.



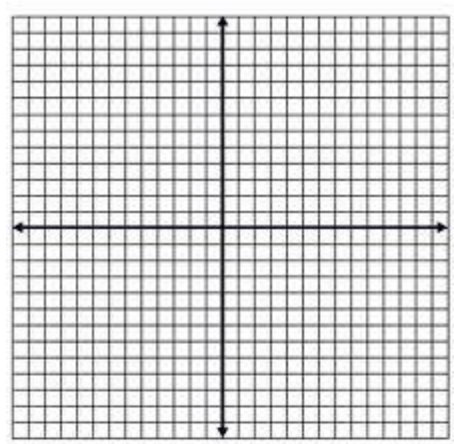
4. Using a calculator, find the volume of the solid generated by revolving about the line $y = 8$ the region bounded by the graph of $y = x^2 + 4$, the line $y = 8$.



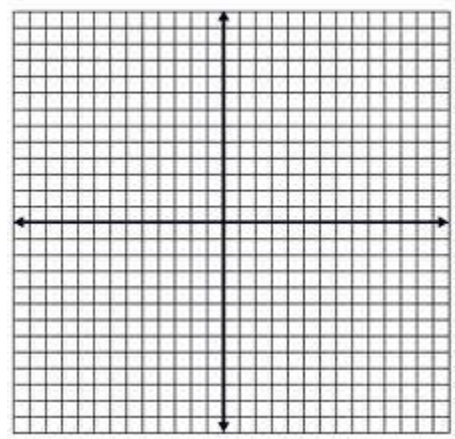
5. Find the volume of the solid of revolution generated by revolving the region bounded by $y = \frac{1}{4}x^2, x = 2, and\ y = 0$ about the $x - axis$.



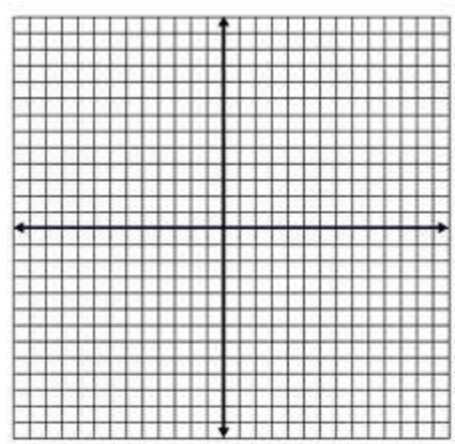
6. Find the volume of the solid of revolution generated by revolving the region bounded by $y = 2x^2, y = 0$, and $x = 2$ about the $x - axis$.



7. Find the volume of the region bounded by $y = x^2 - 2$, $y = -2$, and $x = 2$ if it is rotated around line $y = -2$.



8. The region in the first quadrant bounded by the graph of $y = secx, x = \frac{\pi}{4}$, and the axes is rotated about the $x - axis$. What is the volume of the solid generated?



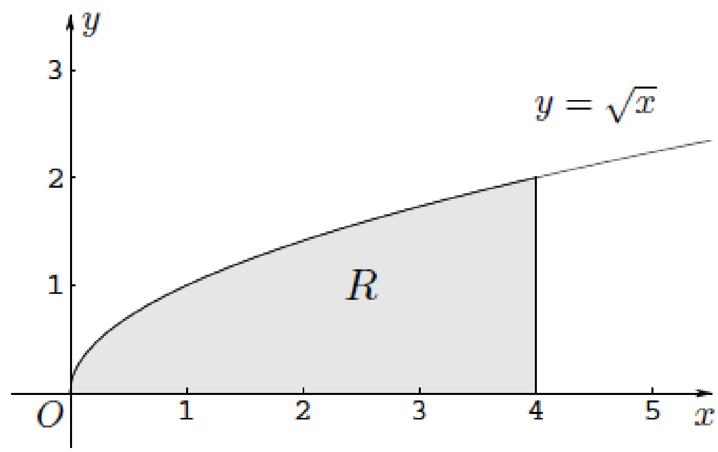
Answer Key:

1. 8π
2. π
3. 18π
4. $\frac{512}{15}\pi$
5. $\frac{2}{5}\pi$
6. $\frac{128}{5}\pi$
7. $\frac{32}{5}\pi$
8. π

Warm Up #7:

Let R be the region bounded by the x —axis , the graph of $y = \sqrt{x}$, and the line $x = 4$.

(a) Find the area of the region R.
(b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
(c) Find the volume of the solid generated when R is revolved around the x —axis.
(d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x —axis they generate solids with equal volumes. Find the value of k.



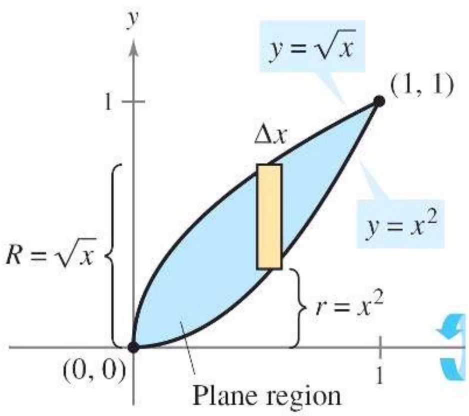
Now Let's Try Some Washers:

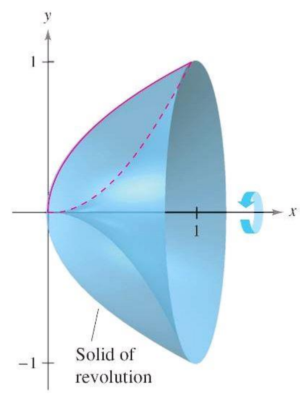
When the region being revolved is not in contact with the axis of revolution, we cannot generate disks. Instead we get a shape known as a washer, which is a disk with a hole in it.

To find the volume we subtract the area of the inner circle from the area of the outer circle.

Example #3:

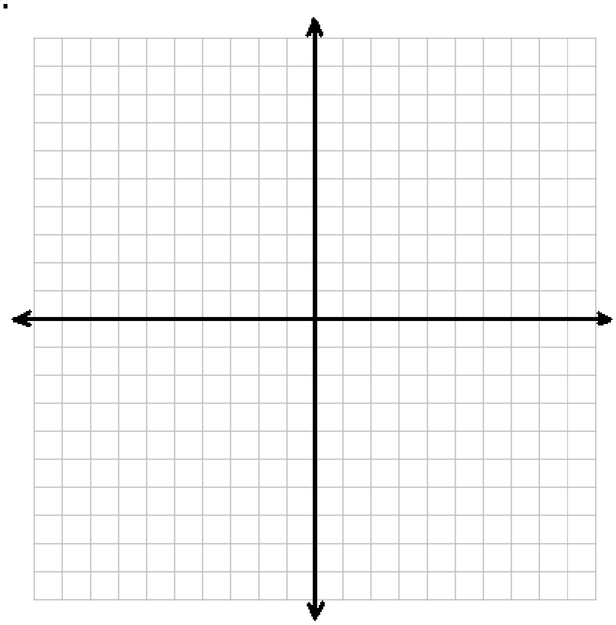
Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the $x - axis$ as shown below.





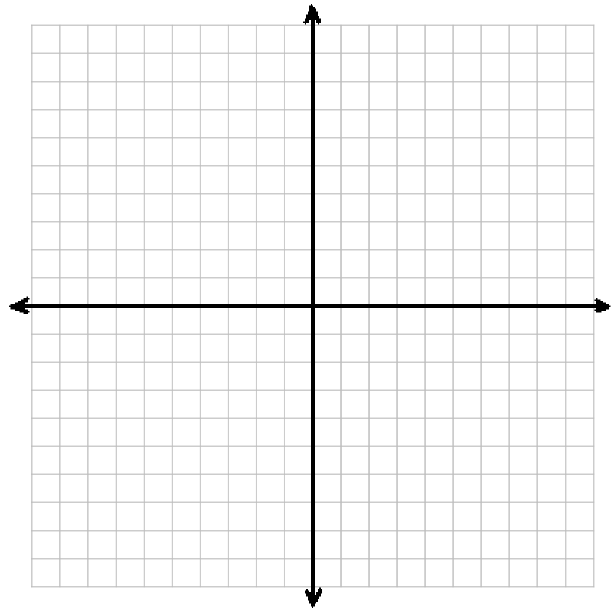
Example #4:

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = e^{-x}$, $y = x + 1$, and $x = 3$ about the line $y = 4$.



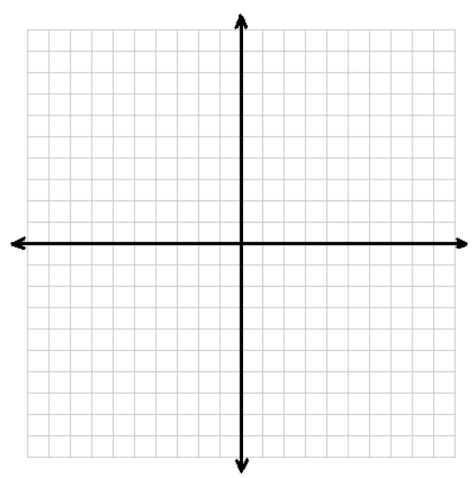
Example #5:

Find the volume of the solid of revolution generated by revolving the region bounded by $y = 2x^2, y = 0$, and $x = 2$ about the $y - axis$.

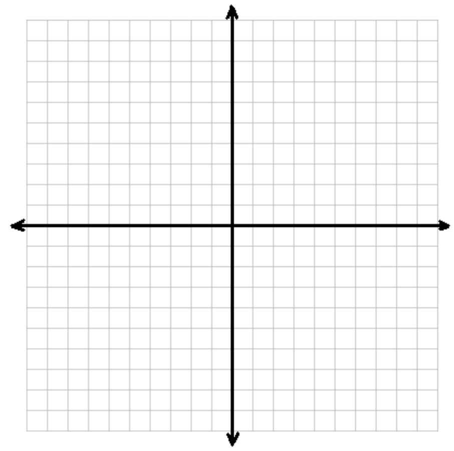


Problem Set #7:

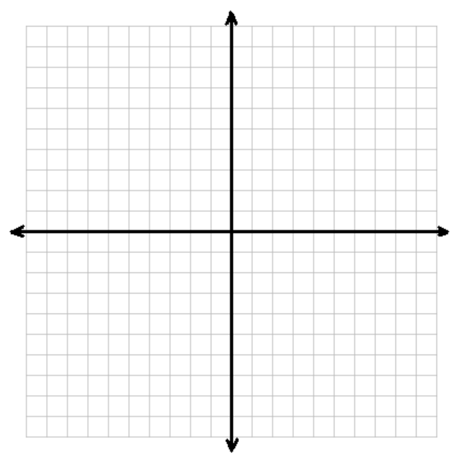
9) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \frac{1}{4}x^2$ and $y = 5 - x^2$ about the $x - axis$.



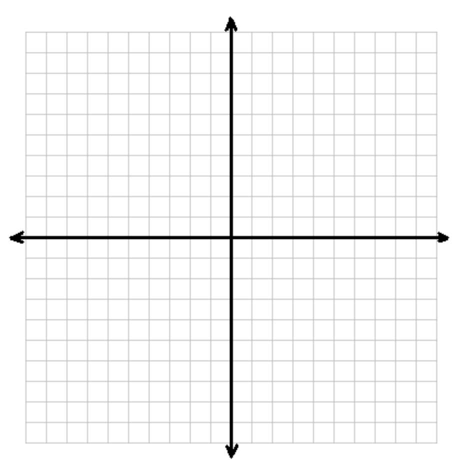
10) Find the volume of the solid of revolution generated by revolving the region bounded by the graphs of $y = 6 - 2x - x^2$ and $y = x + 6$ about the line $y = 3$.



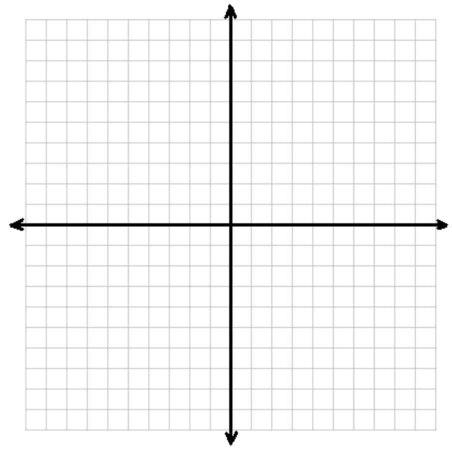
11) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x$ and $y = x^2 - 4x + 4$ about the line $y = 4$.



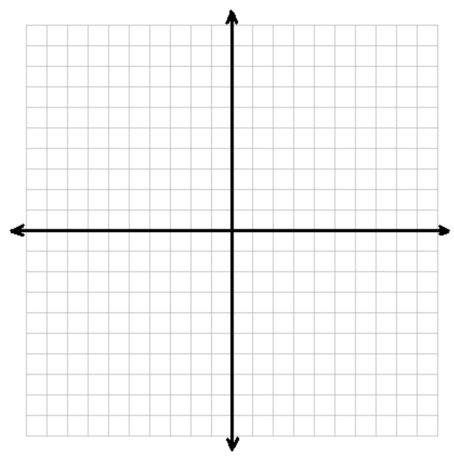
12) Find the volume of the solid of revolution generated by revolving the region bounded by the graphs of $y = 3x,\ y = 12 - 3x$ and $y = 0$ about the $y - axis$.



13) Find the volume of the solid of revolution generated by revolving the region bounded by the graphs of $y = x^3$ and $y = x$ in the first quadrant about the line $y = -2$.



14) Find the volume of the solid of revolution generated by revolving the region bounded by the graphs of $y = 4 - x^2$, $x = 0$, and $y = 0$ about the line $x = 2$.



Answers Key: 9) $\frac{176}{3}\pi$ 10) $\frac{108}{5}\pi$ 11) $\frac{108}{5}\pi$ 12) 48π

13) $\frac{25}{21}\pi$ 14) $\frac{40}{3}\pi$