

Unit #2: Methods of Integration

Topic: Integration by Parts Formula

Objective: *SWBAT evaluate an integral involving the product of two functions by using the integration by parts formula.*

Warm Up #6:

a) *What are some of the common mistakes that could occur when answering the following question?*

“If $f(x) = 2x\sin x - x^2\cos x + 2\cos x + 3$, find $f'(x)$.”

b) *What is the actual answer to the problem given above?*

c) *Why couldn't we use a u-sub to integrate the answer found in part b?*

Integration by Parts is the “product rule” for integration. Just like integration by u-substitution, it is intended to split one complicated function into two easier ones.

Substitution looks for _____;
integration by parts looks for _____ of two unrelated functions.

For example: $\int x^2 \sin x dx$

If u and v are functions of x and have continuous derivatives, then we can integrate using the formula:



The goal is to create an expression that contains an integral that is much easier to integrate than the original. When choosing u and dv , we want to find a u that will be simplified after we take its derivative, and a dv that won't be too complex after integrating it.

When deciding what to choose for u , remember L I P E T.

L - logarithmic function
I - inverse trig function
P - polynomial function
E - exponential function
T - trigonometry function

This is usually the preference order in which you would want to choose u .

Ok, Now let's try to integrate

$$\int u dv = uv - \int v du$$

Example #1: $\int x^2 \sin x dx$

Example #2: $\int \ln x dx$

Example #3: $\int 4 \sin^{-1} x dx$

Problem Set #6: Integrate each of the following using the integration by parts formula.

1) $\int x^5 \ln x dx$

2) $\int x \csc^2 x dx$

3) $\int (x^2 + 1)e^{-x} dx$

4) $\int 2xe^{4x} dx$

5) $\int x^2 \cos\left(\frac{x}{2}\right) dx$

6) $\int \frac{\ln x}{x^2} dx$

7) $\int 3x^2 e^{2x} dx$

8) $\int 3 \arctan x dx$

9) $\int x \sec^2 x dx$

10) $\int \sqrt{x} \ln x dx$

Answer Key

1) $\frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C$	2) $-x \cot x + \ln \sin x + C$
3) $-e^{-x}(x^2 + 1) - 2xe^{-x} - 2e^{-x} + C$	4) $\frac{1}{2}xe^{4x} - \frac{1}{8}e^{4x} + C$
5) $2x^2 \sin \frac{1}{2}x + 8x \cos \frac{1}{2}x - 16 \sin \frac{1}{2}x + C$	6) $-\frac{\ln x }{x} - \frac{1}{x} + C$
7) $\frac{3}{2}x^2 e^{2x} - \frac{3}{2}x e^{2x} - \frac{3}{2}e^{2x} + C$	8) $3x \arctan x - \frac{3}{2} \ln 1 + x^2 + C$
9) $x \tan x + \ln \cos x + C$	10) $\frac{2}{3}x^{3/2} \ln x - \frac{4}{9}x^{3/2} + C$