

Unit #5: Limits

Topic: Limits at Infinity

Objective: SWBAT find limits at infinity by using horizontal asymptotes.

## Warm Up #6:

$$\text{If } f(x) = \begin{cases} x + 5 & \text{for } x < 0 \\ 2 & \text{for } 0 \leq x < 3 \\ x^2 - 7 & \text{for } x \geq 3 \end{cases}, \text{ find}$$

a)  $\lim_{x \rightarrow 0} f(x) =$

b)  $\lim_{x \rightarrow 3} f(x) =$

## Limits at Infinity

A limit at infinity is the value a function is approaching as  $x$  gets infinitely large or infinitely small.



In order to find the limits of a function as  $x$  approaches positive or negative infinity we need to look at the horizontal asymptotes.

If \_\_\_\_\_ is a horizontal asymptote of  $f(x)$  then  $\lim_{x \rightarrow \pm\infty} f(x) =$  \_\_\_\_\_.

What happens to an expression as  $x$  approaches infinity?

Example #1: Find

a)  $\lim_{x \rightarrow \infty} x^2 + 3$

b)  $\lim_{x \rightarrow \infty} \left(2 + \frac{1}{x}\right)$

c)  $\lim_{x \rightarrow -\infty} \frac{x^3 - 10}{8}$

Evaluate each of the following using your calculator:

Given the function  $f(x) = \frac{x^3 + x^2 + 1}{2x^3}$ , find

1) $f(10) =$	2) $f(100) =$	3) $f(1000) =$
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*What do you notice about the values of  $f(x)$  as  $x$  increases?*

*How can you see this in the function itself?*

*Example #2: Find*

a) $\lim_{x \rightarrow -\infty} \frac{(x+1)^2}{3x^2}$	b) $\lim_{x \rightarrow -\infty} \left(7 - \frac{5x}{x^2+1}\right)$	c) $\lim_{x \rightarrow \infty} \frac{x^3+4x-8}{x^2-x-6}$
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*Problem Set #6: Find each of the following limits.*

1) $\lim_{x \rightarrow \infty} \frac{4}{2x+3}$	2) $\lim_{x \rightarrow -\infty} \left(\frac{2}{x} + 1\right) \left(\frac{5x^2-1}{x^2}\right)$
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$$3) \lim_{x \rightarrow \infty} \frac{2e^x + 5}{6}$$

$$4) \lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 + 1}}{4x + 2}$$

$$5) \lim_{x \rightarrow \infty} \left( \frac{2x}{x+1} \right) \left( \frac{3^x - 1}{2} \right)$$

$$6) \lim_{x \rightarrow -\infty} \frac{3x^4 + 1}{2}$$

$$7) \lim_{x \rightarrow \infty} \left[ 3 + \frac{2x^2}{(x+5)^2} \right]$$

$$8) \lim_{x \rightarrow \infty} \frac{2x^2 - 6}{(x-1)^2}$$

$$9) \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}}$$

$$10) \lim_{x \rightarrow \infty} \frac{2+x}{2-x}$$

$$11) \lim_{x \rightarrow -\infty} \frac{7}{e^x + 2e}$$

$$12) \lim_{x \rightarrow \infty} \left( \frac{2}{3} - \frac{4}{(x-12)^3} \right)$$

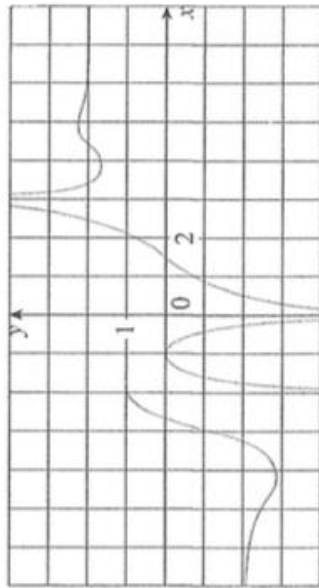
# Homework

4. For the function  $g$  whose graph is given, state the following.

(a)  $\lim_{x \rightarrow -\infty} g(x)$       (b)  $\lim_{x \rightarrow \infty} g(x)$

(c)  $\lim_{x \rightarrow -3} g(x)$       (d)  $\lim_{x \rightarrow 0} g(x)$

(e)  $\lim_{x \rightarrow -2^+} g(x)$       (f) The equations of the asymptotes



5-10 Sketch the graph of an example of a function  $f$  that satisfies all of the given conditions.

5.  $f(0) = 0$ ,  $f(1) = 1$ ,  $\lim_{x \rightarrow 2} f(x) = 0$ ,  $f$  is odd

6.  $\lim_{x \rightarrow 0^+} f(x) = \infty$ ,  $\lim_{x \rightarrow 0^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow \infty} f(x) = 1$ ,  
 $\lim_{x \rightarrow -\infty} f(x) = 1$

7.  $\lim_{x \rightarrow -2} f(x) = -\infty$ ,  $\lim_{x \rightarrow \infty} f(x) = \infty$ ,  $\lim_{x \rightarrow -\infty} f(x) = 0$ ,  
 $\lim_{x \rightarrow 0^+} f(x) = \infty$ ,  $\lim_{x \rightarrow 0^-} f(x) = -\infty$

8.  $\lim_{x \rightarrow -2} f(x) = \infty$ ,  $\lim_{x \rightarrow \infty} f(x) = 3$ ,  $\lim_{x \rightarrow -\infty} f(x) = -3$

9.  $f(0) = 3$ ,  $\lim_{x \rightarrow 0^+} f(x) = 4$ ,  $\lim_{x \rightarrow 0^-} f(x) = 2$ ,  
 $\lim_{x \rightarrow \infty} f(x) = -\infty$ ,  $\lim_{x \rightarrow 4^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow 4^+} f(x) = \infty$ ,  
 $\lim_{x \rightarrow -\infty} f(x) = 3$

10.  $\lim_{x \rightarrow \infty} f(x) = -\infty$ ,  $\lim_{x \rightarrow 2} f(x) = 2$ ,  $f(0) = 0$ ,  $f$  is even

13-14 Evaluate the limit and justify each step by indicating the appropriate properties of limits.

13.  $\lim_{x \rightarrow \infty} \frac{3x^2 - x + 4}{2x^2 + 5x - 8}$

14.  $\lim_{x \rightarrow \infty} \sqrt{\frac{12x^3 - 5x + 2}{1 + 4x^2 + 3x^3}}$

15-36 Find the limit.

15.  $\lim_{x \rightarrow \infty} \frac{1}{2x + 3}$

16.  $\lim_{x \rightarrow \infty} \frac{3x + 5}{x - 4}$

17.  $\lim_{x \rightarrow \infty} \frac{1 - x - x^2}{2x^2 - 7}$

18.  $\lim_{y \rightarrow \infty} \frac{2 - 3y^2}{5y^2 + 4y}$

19.  $\lim_{x \rightarrow \infty} \frac{x^3 + 5x}{2x^3 - x^2 + 4}$

20.  $\lim_{t \rightarrow \infty} \frac{t^2 + 2}{t^3 + t^2 - 1}$

21.  $\lim_{u \rightarrow \infty} \frac{4u^4 + 5}{(u^2 - 2)(2u^2 - 1)}$

22.  $\lim_{x \rightarrow \infty} \frac{x + 2}{\sqrt{9x^2 + 1}}$

23.  $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

24.  $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

25.  $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x)$

26.  $\lim_{x \rightarrow \infty} (x + \sqrt{x^2 + 2x})$

27.  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + ax} - \sqrt{x^2 + bx})$

28.  $\lim_{x \rightarrow \infty} \cos x$

29.  $\lim_{x \rightarrow \infty} \frac{x + x^3 + x^5}{1 - x^2 + x^4}$

30.  $\lim_{x \rightarrow \infty} \sqrt{x^2 + 1}$

31.  $\lim_{x \rightarrow \infty} (x^4 + x^5)$

32.  $\lim_{x \rightarrow \infty} \frac{x^3 - 2x + 3}{5 - 2x^2}$

33.  $\lim_{x \rightarrow \infty} \frac{1 - e^x}{1 + 2e^x}$

34.  $\lim_{x \rightarrow \infty} \tan^{-1}(x^2 - x^4)$

35.  $\lim_{x \rightarrow \infty} (e^{-2x} \cos x)$

36.  $\lim_{x \rightarrow (\pi/2)^+} e^{\sin x}$