

Unit #9: Rates of Change and Particle Motion

Topic: Velocity and Rates of Changes

Objective: SWBAT will be able to interpret the derivative in order to model and explain particle motion numerically.

Warm Up #6:

A point moves along the x – $axis$ so that its position x at time t is specified by the function $x(t) = t^3 - 9t - 9$. Determine the following:

- (a) the time intervals on which the point is moving to the right and to the left.

- (b) the time intervals on which the point has positive and negative acceleration.

- (c) the time intervals when the particle is speeding up and slowing down.

- (d) the acceleration at times when the velocity is zero.

- (e) the average velocity over the time interval $[0,6]$.

Sometimes we aren't given an equation or a graph, but rather a table of discrete values. In these cases, we are at the mercy of the limited information given to us.

Numerically:

- 1) The values of the coordinate s of a particle moving smoothly and continuously along a line for various values of t are given below:

t (sec)	0	0.5	1	1.5	2	2.7	3	3.6	4
s (ft)	40.0	35.0	30.2	36.0	48.2	45.0	38.2	16.0	0.2

- (a) What is the displacement of the particle during the given 4-second interval?
- (b) How many times does the particle change direction for $t \in [0,4]$ seconds? Justify.
- (c) What is the particle's average velocity for $0 \leq t \leq 0.5$ seconds?
- (d) Estimate the velocity at each of the following:
- (i) at $t = 0.5$ sec (ii) at $t = 2.7$ sec (iii) at $t = 3.5$ sec

2) The data in the table below give selected values for the velocity, in meters/minute, of a particle moving along the x – axis, The velocity v is a differentiable function of time t .

Time t (min)	0	2	5	7	10	12
Velocity $v(t)$ (meters/min)	-3	2	3	-5	7	5

- (a) At $t = 0$, is the particle moving to the right or to the left? Explain your answer.
- (b) Find the acceleration of the particle $t = 1$ seconds?
- (c) Is there a time during the time interval $0 \leq t \leq 12$ minutes when the acceleration is negative? Explain your answer.
- (d) Is there a time during the time interval $0 \leq t \leq 12$ minutes when the particle is at rest? Explain your answer.
- (e) Use the data in the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer and indicate units of measure.

3) The velocity of a particle moving along the x – axis is modeled by a differentiable function v , where the position x is measured in meters, and the time t is measured in seconds. Selected values of $v(t)$ are given in the table below. The particle is at position $x = 7$ meters when $t = 0$ seconds.

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	-10	-8	-4	7

- Is the distance increasing or decreasing when $t = 8$? Explain your answer.
- Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.
- For $0 \leq t \leq 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.
- Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.