

Unit #9: Rates of Change and Particle Motion

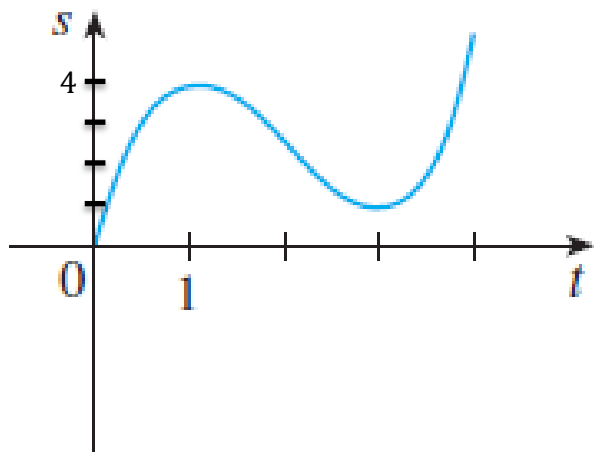
Topic: Position, Velocity, and Acceleration Free Response Problems

Objective: SWBAT will be able to interpret the derivative in order to model and explain particle motion.

Warm Up #7:

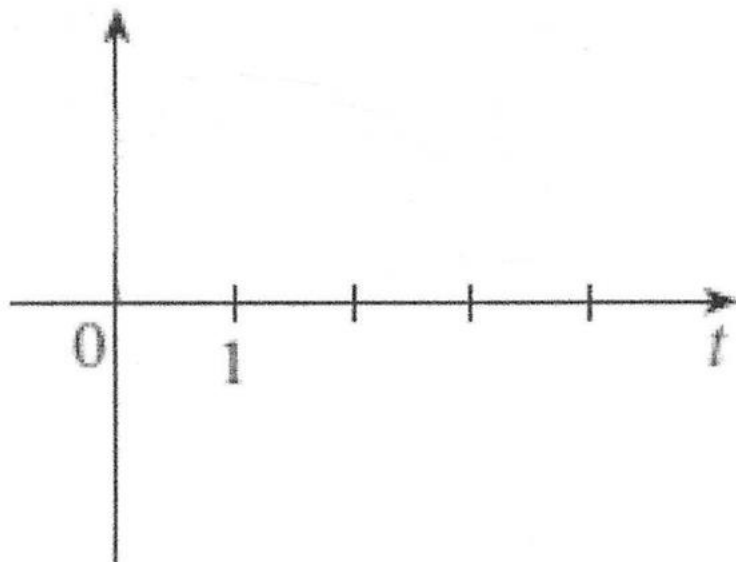
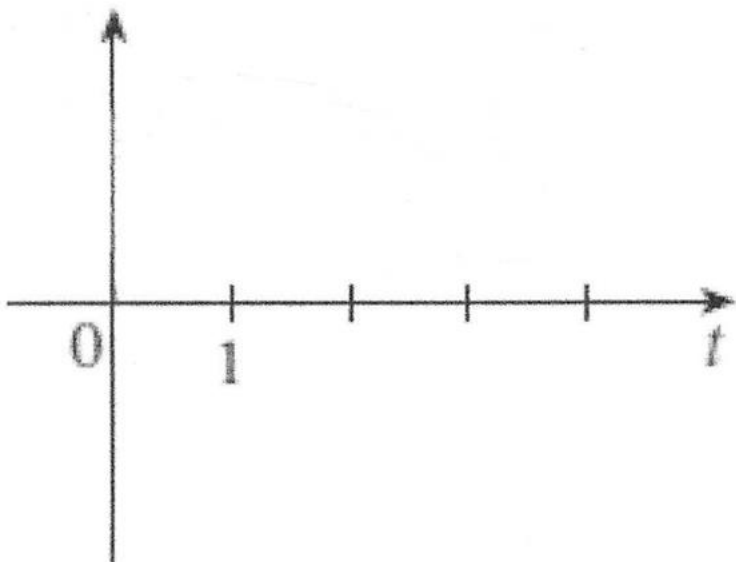
The graph below shows the position $s(t)$ of a particle moving along a horizontal line.

a) Describe the motion of the particle for $0 \leq t \leq 4$.



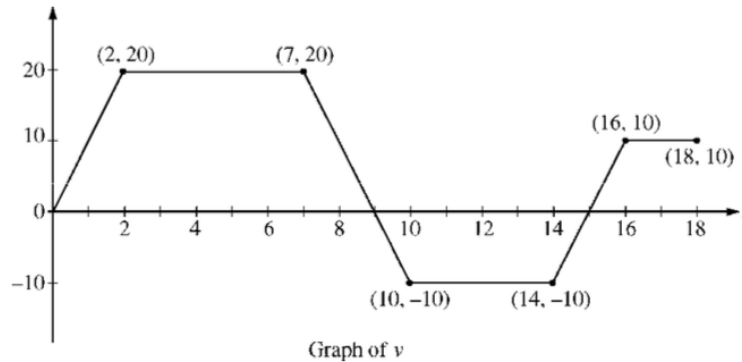
b) Sketch the graph of $s'(t)$.

c) Sketch the graph of $s''(t)$.



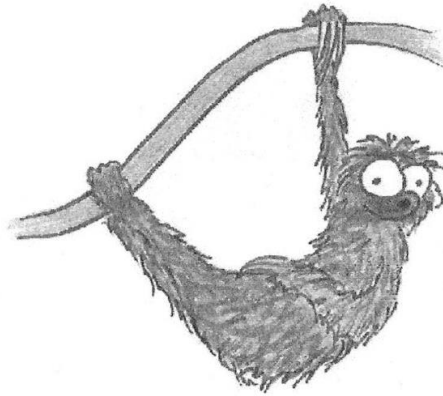
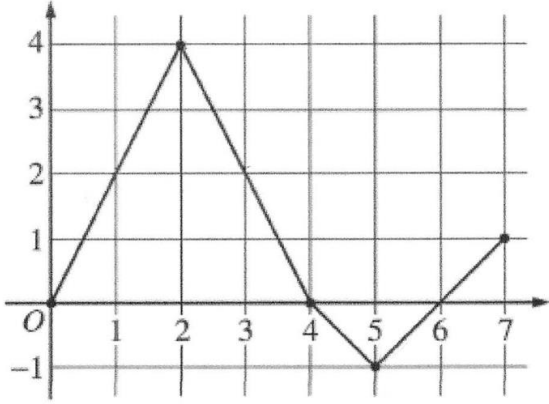
Practice Problems:

- 1) A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B . For $0 \leq t \leq 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph below.



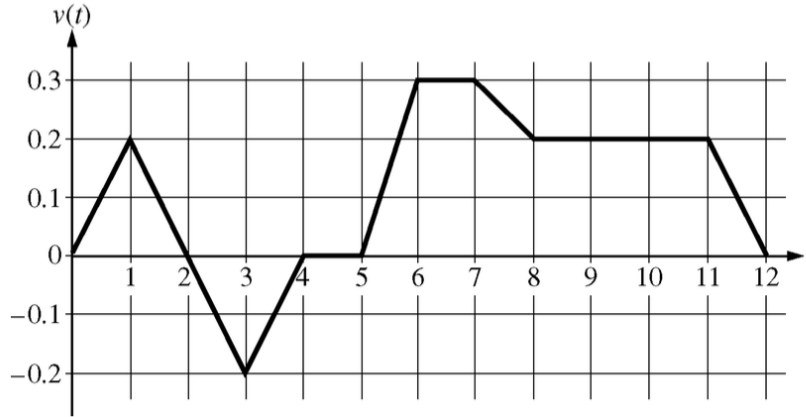
- At what time in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.
- Describe the motion of the squirrel in the interval $0 < t < 18$. Explain your reasoning.
- When is distance increasing? decreasing?
- When is the squirrel speeding up? slowing down?
- During what time interval is the speed the greatest?

- 2) A three-toed sloth is hanging vertically from its tail on a horizontal tree branch, high in the sky. The graph below describes the horizontal movement, in inches, of the sloth along the branch as it moves from left to right, with zero corresponding to the middle of the branch. Time is measured in hours along the x - axis.



- What is the displacement of the sloth during the first three hours?
- What is the sloth's average velocity during the first three hours?
- What is the sloth's speed at hour 3? In which direction is he moving?
- At what time(s) does the sloth change directions?
- At what time is the sloth the furthest from the center of the branch?
- How many total inches did the sloth move during the 7 hour period?

- 3) Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \leq t \leq 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown below.



- Describe Caren's movement during the 12 minute interval including when she is moving to the left, to the right, when she is at rest, and when she changes direction.
- During what time intervals is Caren's velocity increasing? decreasing?
- During what time intervals is Caren speeding up? slowing down?
- When is Caren's speed the greatest?

- 4) A particle moves along the x – $axis$ so that its position at time t , $0 \leq t \leq 5$, is given by $x(t) = t^3 - 6t^2 + 9t - 2$.
- (a) Find the total distance traveled by the particle.
 - (b) Find the average velocity of the particle over the time interval $0 \leq t \leq 5$.
 - (c) Find the minimum acceleration of the particle.
 - (d) Find the velocity of the particle when the acceleration equals zero.

Warm Up #8:

CALCULATOR ALLOWED

The horizontal position of a particle moving on the line $y = 2$, is given by $x(t) = 2t^3 - 13t^2 + 22t - 5$ where t is time in seconds.

- (a) Describe the motion of the particle for $t \geq 0$.
- (b) When does the particle speed up? slow down?
- (c) When does the particle change direction?
- (d) When is the particle at rest?
- (e) Describe the velocity and speed of the particle.
- (f) When is the particle at the point $(5,2)$?

Practice Problems:

5) A particle moves along the x -axis so that at time t its position is given by $x(t) = t^3 - 6t^2 + 9t + 11$.

- (a) What is the velocity of the particle at $t = 0$?
- (b) During what time intervals is the particle moving to the left?
- (c) What is the total distance traveled by the particle from $t = 0$ to $t = 2$?
- (d) For what values of t is the speed increasing?

- 6) A particle moving along the x - axis so that its position at any time $t \geq 0$ is given by $s(t) = t^3 - t^2 - t + 3$.
- (a) Find the velocity of the particle at any time t .
 - (b) For what values of t , $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0,3]$?
 - (c) Find the displacement of the particle from time $t = 0$ to time $t = 3$.
 - (d) Find the total distance the particle traveled from time $t = 0$ to time $t = 3$.