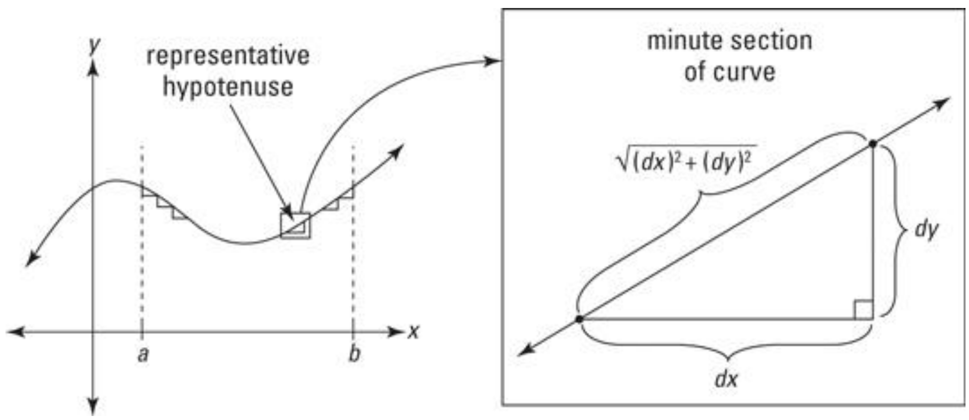
Unit #4: Area and Volume
Topic: Arc Length
Objective: SWBAT find arc length by using integration.

Most functions have a curve to them so we need a way to find the length of the curve.

For example, think of the sine wave. We already know how to find the area under the curve, but what if we wanted to know how long is the curve itself. If you straightened the curve out like a piece of string along the positive x-axis with one end at zero, where would the other end be?

We have seen how integration can be used to approximate areas and volumes. We can use the same technique to find the length of a curve by using line partitions and the distance formula.



If the derivative of a function $y = f(x)$ is continuous on the interval $a \le x \le b$, then we call this a ***smooth*** curve and to find the length of the curve is given by

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

If the derivative of a function $x = g(y)$ is continuous on the interval $c \le y \le d$, then *the length of the curve is given by*

$$s = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Example #1:

Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1$ for $0 \leq x \leq 1$.

Example #2:

Find the length of the curve $x = \frac{2}{3}(y-1)^{3/2}$ for $1 \leq y \leq 4$.

Example #3:

The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1 + 9x^4}\,dx$. If the curve contains the point (1, 6), which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$

(B) $y = 5 + x^3$

(C) $y = 6 + x^3$

(D) $y = 6 - x^3$

(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$

Problem Set #8:

1. Find the length of the curve $y = \frac{1}{6}x^3 + \frac{1}{2}x^{-1}$ from $x = 1\ from\ x = 2$.

2. What is the length of the curve $x = y^2 - 2$ for $\sqrt{3} \leq y \leq 3$?

3. Find the length of the arc of $y = \cos(2x)$ from $x = 0\ \ to\ x = {}^{\pi}/_{4}$.

4. Find the length of the curve given by $y^2 + 2y = 2x + 1$ from $(-1, -1)\ to\ (7{,}3)$.

5. The length of the curve $y = \ln \sec x$ from $x = 0$ to $x = b$, where $0 < b < \frac{\pi}{2}$, may be expressed by which of the following integrals?

(A) $\int_0^b \sec x \, dx$

(B) $\int_0^b \sec^2 x \, dx$

(C) $\int_0^b (\sec x \tan x) \, dx$

(D) $\int_0^b \sqrt{1 + (\ln \sec x)^2} \, dx$

(E) $\int_0^b \sqrt{1 + \left(\sec^2 x \tan^2 x\right)} \, dx$

6. Find the length of the curve $y = x^2 + 3$ for $1 \le x \le 3$.

7. What is the length of the curve produced by the function $x = 4(3 + y)^2$ from $1 \le y \le 4$?

8. Find the length of the arc of $y = (x-1)^{3/2}$ from $x = 1 \; to \; x = 5$.

Answers: 1. 17/12 2. 6.136 3. 1.318 4. 9.294 5. A 6. 8.611 7. 130.957 8. 9.07