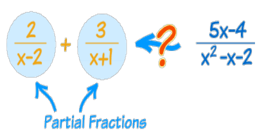
Unit: Methods of Integration
Topic: Integration with Partial Fractions
Objective: SWBAT integration rational functions by using a partial fraction decomposition.

Warm Up #9:

If $\int x^2 cos x dx = h(x) - \int 2x sin x dx$, then what is $h(x)$?



Partial fraction decomposition can be a powerful tool to use when antidifferentiating rational functions.

A difficult integral involving a rather complex rational expression can be made simpler by breaking the expression into pieces using partial fractions.

Example #1: Evaluate $\int \frac{8x+7}{x^2+x-2} dx$

Example #2: Evaluate $\int \frac{x-13}{2x^2-7x+3}\,dx$

Problem Set #9: Evaluate each of the following using a partial fraction decomposition.

1) $\int \frac{5x-3}{x^2-2x-3}\,dx$	2) $\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx$

3) $\int \frac{2x+16}{x^2+x-6}\,dx$	4) $\int \frac{1-3x}{3x^2-5x+2}\,dx$
5) $\int \frac{x+5}{x^2+x-2}\,dx$	6) $\int \frac{5x+14}{x^2+7x}\,dx$

7) $\int \frac{x^2-x+4}{x^3-3x^2+2x}\,dx$	8) $\int \frac{6x^2-8x-4}{(x^2-4)(x-1)}\,dx$

Answer Key:

1) $ln(\lvert x-3\rvert^3(x+1)^2)+C$	2) $\frac{1}{2}ln\lvert x\rvert+\frac{1}{10}ln\lvert 2x-1\rvert-\frac{1}{10}ln\lvert x+2\rvert+C$
3) $ln\left(\frac{(x-2)^4}{(x+3)^2}\right)+C$	4) $ln\left(\frac{\lvert 3x-2\rvert}{(x-1)^2}\right)+C$
5) $2ln\lvert x-1\rvert-ln\lvert x+2\rvert+C$	6) $ln(x^2\lvert x+7\rvert^3)+C$
7) $ln\left(\frac{x^2\lvert x-2\rvert^3}{(x-1)^4}\right)+C$	8) $ln(\lvert x-2\rvert\lvert x+2\rvert^3(x-1)^2)+C$

Warm Up #10:

Divide the following expression using long division: $\frac{2x^3+x^2-7x+7}{x^2+x-2}$

For integration of rational functions, it is **ALWAYS** necessary to perform polynomial division first, if possible.

If the integrand is a rational function for which the degree of the numerator is greater than or equal to the degree of the denominator we must use long division to rewrite the integrand as the following.

$$\int \text{quotient} + \frac{remainder}{divisor}$$

Then, if necessary, we may need to use partial fractions.

Example #3: Evaluate $\int \frac{2x^3+x^2-7x+7}{x^2+x-2}\,dx$

Problem Set #10: Evaluate each of the following integrals.

9) $\int \frac{x^2-x+1}{x^2-x}\,dx$

10) $\int \frac{2x^2+x+3}{x^2-9}\,dx$

11) $\int \frac{9x^3+4x+3}{x^2-1}\,dx$

12) $\int \frac{x^2+2x+9}{x^2+4x-5}\,dx$

13) $\frac{4x^3+2x^2+1}{4x^3-x}\,dx$

14) $\frac{x^2-1}{x^2-16}\,dx$

15) $\frac{3x^3-5x^2-11x+9}{x^2-2x-3}dx$
16) $\frac{x^4+x^3+x^2+1}{x^2+x-2}dx$

Answer Key:

9) $x+ln\left\|\frac{x-1}{x}\right\|+C$	10) $2x+ln\left(\frac{(x-3)^4}{\|x+3\|^3}\right)+C$
11) $\frac{9}{2}x^2+ln((x-1)^8\|x+1\|^5)+C$	12) $x+ln\left(\frac{(x-1)^2}{(x+5)^4}\right)+C$
13) $x+ln\frac{\|2x-1\|}{\|x\|}+\frac{1}{2}ln\|2x+1\|+C$	14) $x+\frac{15}{8}ln\frac{\|x-4\|}{\|x+4\|}+C$
15) $\frac{3}{2}x^2+x+3ln\|x-3\|-3ln\|x+1\|+C$	16) $\frac{1}{3}x^3+3x-\frac{13}{3}ln\|x+2\|+\frac{4}{3}ln\|x-1\|+C$