Midterm Review

Directions: Read each problem carefully and show all work.

No Calculator Allowed, unless otherwise specified.

1. $\int x^2 \cos\left(x^3\right)\, dx =$

2. $\lim_{x\to 0} \frac{1-cosx}{x} =$

3. $\int xe^{2x} dx =$

(A) $\frac{xe^{2x}}{2}-\frac{e^{2x}}{4}+C$ (B) $\frac{xe^{2x}}{2}-\frac{e^{2x}}{2}+C$ (C) $\frac{xe^{2x}}{2}+\frac{e^{2x}}{4}+C$

(D) $\frac{xe^{2x}}{2}+\frac{e^{2x}}{2}+C$ (E) $\frac{x^2e^{2x}}{4}+C$

4. $\int \frac{dx}{(x-1)(x+2)} =$

(A) $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right|+C$ (B) $\frac{1}{3}\ln\left|\frac{x+2}{x-1}\right|+C$ (C) $\frac{1}{3}\ln\left|(x-1)(x+2)\right|+C$

(D) $\left(\ln|x-1|\right)\left(\ln|x+2|\right)+C$ (E) $\ln\left|(x-1)(x+2)^2\right|+C$

5. $\int \ln(5x)dx$

6. $$\int \frac{2x}{(x+2)(x+1)}\,dx =$$

(A) $\ln|x+2| + \ln|x+1| + C$

(B) $\ln|x+2| + \ln|x+1| - 3x + C$

(C) $-4\ln|x+2| + 2\ln|x+1| + C$

(D) $4\ln|x+2| - 2\ln|x+1| + C$

(E) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$

7. $$f(x) = \begin{cases} \frac{x^2-9}{x-3} & if \;\; x \neq 3 \\ 1 & if \;\; x = 3 \end{cases}$$

Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 3$.

II. f is continuous at $x = 3$.

III. f is differentiable at $x = 3$.

(A) I only (B) I and II only (C) III only (D) II only (E) I, II, and III

8. $\frac{d}{dt}\int_0^{2t} \frac{1-cosx}{x}\,dx$

9. Consider the curve given by $xy^2 - x^3y = 6$.

   a) Find $\frac{dy}{dx}$.

   b) Find all the points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

   c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

10. Find the average value of $f(x) = 2x - x^2$ on the interval [0,2].

11. What is the slope of the polar curve $r = 3 + 2cos\theta$ at $\theta = \frac{\pi}{2}$?

12. **Calculator**

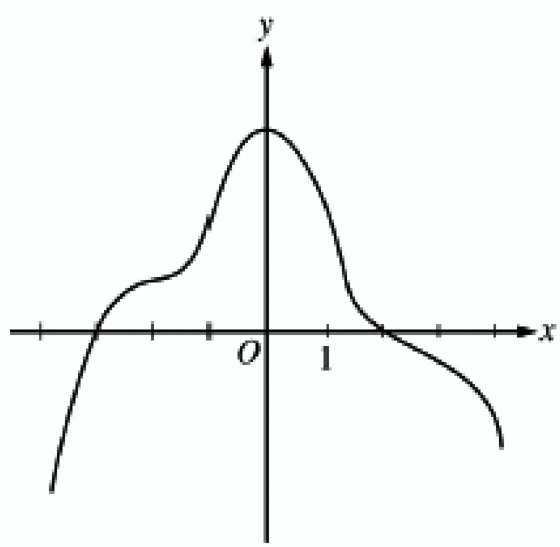
The velocity of a particle moving along the x-axis is given by

$$v(t) = x^4 - 3x^3 - 9x^2 + 23x - 12.$$

How many times does the particle change direction as t increases from $-5\ to\ 5$?

13. The circumference of a circle is increasing at a rate of $\frac{2\pi}{5}$ inches per minute. When the radius is 5 inches, how fast is the area of the circle increasing in square inches per minute?

14.



Graph of f'

The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$.

II. The graph of f has a point of inflection at $x = -2$.

III. The graph of f is concave down for $0 < x < 4$.

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

15. Find the instantaneous rate of change of $f(t) = (2t^3 - 3t + 4)\sqrt{t^2 + 3t + 4}$ at $t = 0$.

16. Let f be the function defined by $f(x) = x^3 - 3x^2$ What is the value of c for which the instantaneous rate of change of f $at\ x = c$ is the same as the average rate of change of f over [0,3]?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

17. Find the equation of the normal line to the curve $y = \sqrt{16 - x}$ at the point (0,4).

18. Find the area of the region enclosed by the polar curve $r = cos(3\theta)$ for $0 \leq \theta \leq \pi$ is

19. Write an integral that can be used to find the length of the curve $y = sin3x$ from $x = 0\ to\ x = 4$.

20. Find the value of each of the following:

a) $\int_1^\infty \frac{1}{x^4}\,dx$.

b) $\int_{-\infty}^\infty \frac{2x}{(x^2+1)^2}\,dx$

c) $\int_0^3 \frac{dx}{x-1}$

21. Which of the following gives the length of the path described by the parametric equations $x(t) = 2 + 3t$ and $y(t) = 1 + t^2$ from $t = 0$ to $t = 1$?

(A) $\int_0^1 \sqrt{1 + \frac{4t^2}{9}}\, dt$

(B) $\int_0^1 \sqrt{1 + 4t^2}\, dt$

(C) $\int_0^1 \sqrt{3 + 3t + t^2}\, dt$

(D) $\int_0^1 \sqrt{9 + 4t^2}\, dt$

(E) $\int_0^1 \sqrt{(2 + 3t)^2 + \left(1 + t^2\right)^2}\, dt$

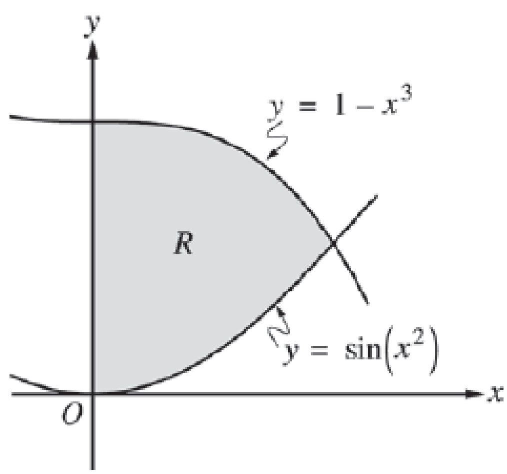
22. If $\int_0^k \frac{x}{x^2+4}\,dx = \frac{1}{2}\ln 4$, where $k > 0$, then $k =$

(A) 0 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{12}$ (E) $\frac{1}{2}\tan\left(\ln\sqrt{2}\right)$

23. If $x = t^2 - 1$ and $y = \ln t$, what is $\frac{d^2y}{dx^2}$ in terms of t?

(A) $-\frac{1}{2t^4}$ (B) $\frac{1}{2t^4}$ (C) $-\frac{1}{t^3}$ (D) $-\frac{1}{2t^2}$ (E) $\frac{1}{2t^2}$

24. **CALCULATOR**



Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$, as shown in the figure above.

(a) Find the area of R.

b) Find the volume of the solid generated when R is revolved about the line $y = -3$.

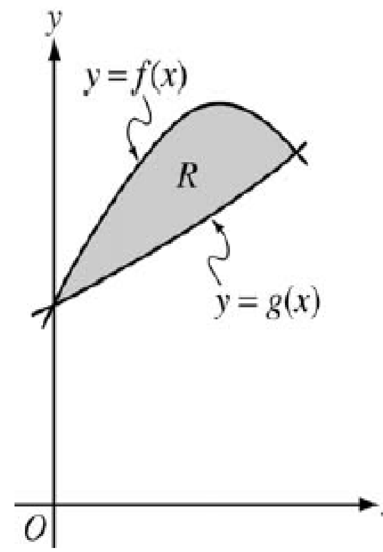
25. **CALCULATOR**

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

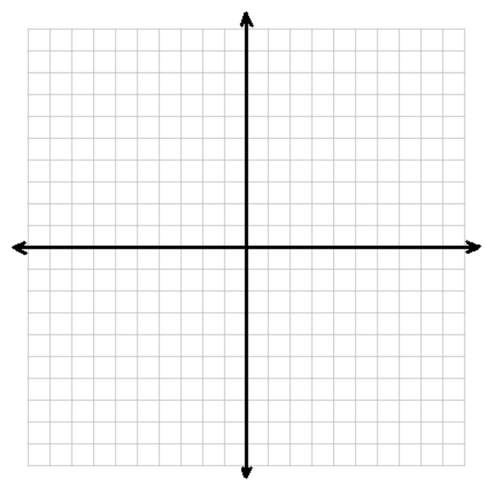
(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.



26. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 2x^2 - x^3$ and $y = 0$ about the $y - axis$.



27. For $0 \leq t \leq 13$, an object travels along an elliptical path given by the parametric equations $x = 3cost$ and $y = 4sint$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

28. **CALCULATOR**

A particle moves in the $xy-plane$ so that its position at any time t is given by $x(t) = t^2$ and $y(t) = sin(4t)$. What is the speed of the particle when $t = 3$?

29. For any time $t \geq 0$, if the position of a particle in the xy-plane is given by $x = t^2 + 1$ and $y = \ln(2t+3)$, then the acceleration vector is

(A) $\left(2t, \dfrac{2}{(2t+3)}\right)$ (B) $\left(2t, \dfrac{-4}{(2t+3)^2}\right)$ (C) $\left(2, \dfrac{4}{(2t+3)^2}\right)$

(D) $\left(2, \dfrac{2}{(2t+3)^2}\right)$ (E) $\left(2, \dfrac{-4}{(2t+3)^2}\right)$

30. The position of a particle moving in the $xy-plane$ is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

(A) -1 only (B) 0 only (C) 2 only (D) -1 and 2 only (E) -1, 0, and 2

31. For $0 \leq t \leq 3$, an object moving along a curve in the xy −plane has position $(x(t), y(t))$ with $\frac{dx}{dt} = \sin(t^3)$ and $\frac{dy}{dt} = 3\cos(t^2)$. At time $t = 2$, the object is at position (4,5). Find the position of the object at time $t = 3$.

32. An object moving along a curve in the $xy - plane$ is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = arctan\left(\frac{t}{1+t}\right) \; and \; \frac{dy}{dt} = ln(t^2 + 1)$$

for $t \geq 0$. At time $t = 0$, the object is at position (-3,-4).

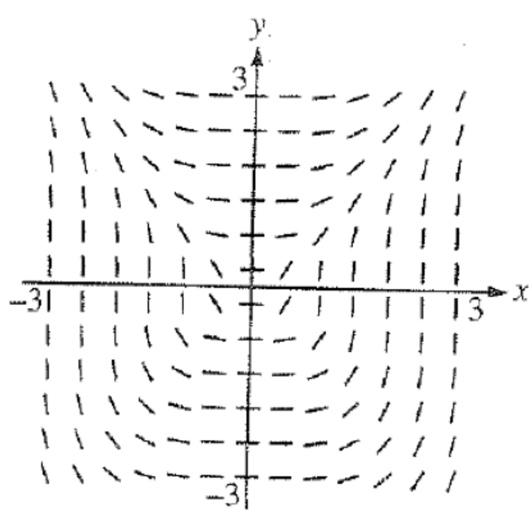
(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

33.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = \frac{x^2}{y^2}$ (C) $\frac{dy}{dx} = \frac{x^3}{y}$ (D) $\frac{dy}{dx} = \frac{x^2}{y}$ (E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

34. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t\to\infty} M(t)$?

(A) 50 (B) 200 (C) 500 (D) 1000 (E) 2000

35. **CALCULATOR**

Ten grizzly bears were introduced to a national park 10 years ago. There are 23 bears in the park at the present time. The park can support a maximum of 100 bears. Assuming a logistic growth model, when will the bear population reach 75?

36. **CALCULATOR**

A hard-boiled egg at 98°C is put in a pan under running 18°C water to cool. After 5 minutes, the egg's temperature is found to be 38°C. How much longer will it take the egg to reach 20°C?

37. **CALCULATOR**

Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton how many tons did the insects destroy during the time interval $7 \le t \le 14$?

38. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$

39. Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the intial condition $f(1) = 4$.

40.

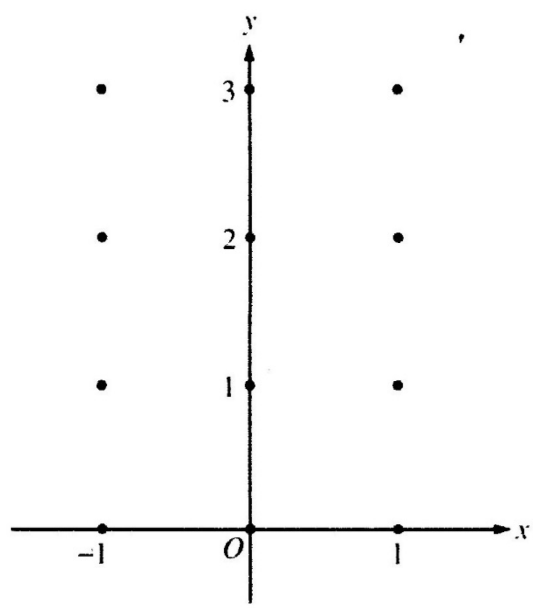
x	2	3	5	8	13
$f(x)$	6	–2	–1	3	9

. The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$, what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

41. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with the initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

42. Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

43. **CALCULATOR**

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

44. **CALCULATOR**

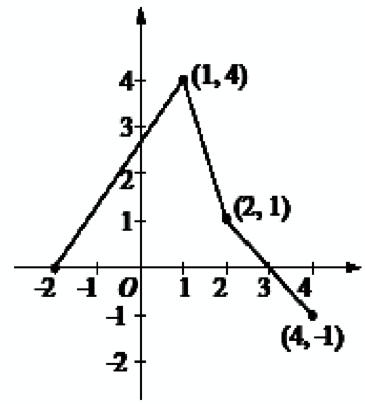
The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

(a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

(c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

45. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.



(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

46. $\int_1^e \frac{x^2+1}{x}\,dx =$

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+1}{2}$ (C) $\frac{e^2+2}{2}$ (D) $\frac{e^2-1}{e^2}$ (E) $\frac{2e^2-8e+6}{3e}$

47. a) $\lim_{x \to 1} \frac{\int_1^x e^{t^2}\, dx}{x^2 - 1}$

b) $\lim_{x \to 0} (1 + 6x)^{\csc x}$

48.

x	0	1	2	3
$f''(x)$	5	0	-7	4

The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

49. On the graph of $y = f(x)$, the slope at any point (x, y) is twice the value of x. If $f(2) = 3$, what is the value of $f(3)$?