Key

Midterm Review

Directions: Read each problem carefully and show all work.

No Calculator Allowed, unless otherwise specified.

U-sub

1. $\int x^2\cos(x^3)\,dx =$ $\frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u + C$

Let $u = x^3$

$du = 3x^2\,dx$

$= \boxed{\frac{1}{3}\sin x^3 + C}$

L'Hopital

2. $\lim_{x\to 0}\frac{1-\cos x}{x} = \frac{0}{0}$ $\lim_{x\to 0}\frac{\sin x}{1} = \boxed{0}$

IBP TABLE

3. $\int xe^{2x}dx =$

(A) $\frac{xe^{2x}}{2}-\frac{e^{2x}}{4}+C$ (B) $\frac{xe^{2x}}{2}-\frac{e^{2x}}{2}+C$ (C) $\frac{xe^{2x}}{2}+\frac{e^{2x}}{4}+C$

(D) $\frac{xe^{2x}}{2}+\frac{e^{2x}}{2}+C$ (E) $\frac{x^2e^{2x}}{4}+C$

DERIV	INT.
$+\ x$	e^{2x}
$-\ 1$	$\frac{1}{2}e^{2x}$
0	$\frac{1}{4}e^{2x}$

PARTIAL

4. $\int\frac{dx}{(x-1)(x+2)} = \int \frac{1}{3(x-1)} - \frac{1}{3(x+2)}\,dx = \frac{1}{3}\int \frac{1}{x-1} - \frac{1}{x+2}\,dx$

(A) $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right|+C$ (B) $\frac{1}{3}\ln\left|\frac{x+2}{x-1}\right|+C$ (C) $\frac{1}{3}\ln|(x-1)(x+2)|+C$

(D) $(\ln|x-1|)(\ln|x+2|)+C$ (E) $\ln|(x-1)(x+2)^2|+C$

$1 = A(x+2) + B(x-1)$

$x=1$ $1 = 3A$ $A = \frac{1}{3}$ $x=-2$ $1=-3B$ $B=-\frac{1}{3}$

$\frac{1}{3}\left[\ln|x-1| - \ln|x+2|\right]$

IBP LIPET

5. $\int \ln(5x)\,dx$

$u = \ln(5x)$

$du = \frac{5}{5x} = \frac{1}{x}\,dx$

$dv = 1\,dx$

$v = x$

$uv - \int v\,du$

$x\ln(5x) - \int x \cdot \frac{1}{x}\,dx$

$\boxed{x\ln(5x) - x + C}$

PARTIAL ~~6.~~

$$\int \frac{2x}{(x+2)(x+1)}\,dx = \frac{A}{x+2} + \frac{B}{x+1} = \int \frac{4}{x+2} - \frac{2}{x+1}\,dx$$

(A) $\ln|x+2| + \ln|x+1| + C$

(B) $\ln|x+2| + \ln|x+1| - 3x + C$

(C) $-4\ln|x+2| + 2\ln|x+1| + C$

(D) $4\ln|x+2| - 2\ln|x+1| + C$

(E) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$

$2x = A(x+1) + B(x+2)$

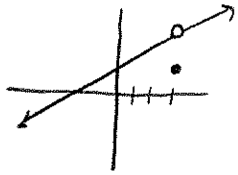
$x = -1 \quad -2 = B$

$x = -2 \quad -4 = -A$

$A = 4$

7.

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & if\ \ x \neq 3 \\ 1 & if\ \ x = 3 \end{cases}$$



Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 3$. ✓

II. f is continuous at $x = 3$. X

III. f is differentiable at $x = 3$. X must be continuous

* Diff $\Rightarrow$ continuity

Cont $\not\Rightarrow$ Diff. (* Think AV graph)

(A) I only (B) I and II only (C) III only (D) II only (E) I, II, and III

✓ 8. $\frac{d}{dt}\int_0^{2t} \frac{1-\cos x}{x}\,dx$ FTC

$\cancel{2}\left(\frac{1-\cos 2t}{\cancel{2}t}\right) = \boxed{\frac{1-\cos 2t}{t}}$

mult by deriv of upper limit

✓ 9. Consider the curve given by $xy^2 - x^3y = 6$.

Implicit Diff a) Find $\frac{dy}{dx}$.

$y^2 + x\cdot 2y\cdot y' - [3x^2y + x^3y'] = 0$

$\frac{y'[\cancel{2xy - x^3}]}{2xy - x^3} = \boxed{\frac{3x^2y - y^2}{2xy - x^3}}$

b) Find all the points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

If $x = 1$ then $y^2 - y - 6 = 0$, $y = 3$ $y = -2$

$(1, 3)$ $(1, -2)$

for $(1,3)$ $\frac{dy}{dx} = \frac{9-9}{6-1} = 0$ $\boxed{y = 3}$

for $(1,-2)$ $\frac{dy}{dx} = \frac{-6-4}{-4-1} = 2$ $\boxed{y + 2 = 2(x-1)}$

c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

Set denom = 0
Plug into curve to solve

$2xy - x^3 = 0$
$x(2y - x^2) = 0$
$\cancel{x=0}$ $y = \frac{1}{2}x^2$

* can't happen $0 - 0 \neq 6$ in equatn

Plug in here $xy^2 - x^3y = 6$

$x(\frac{1}{2}x^2)^2 - x^3(\frac{1}{2}x^2) = 6$

$\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$

$-\frac{1}{4}x^5 = 6$

$x^5 = -24$

$\boxed{x = \sqrt[5]{-24}}$

✓ 10. Find the average value of $f(x) = 2x - x^2$ on the interval $[0,2]$.

$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2}\int_0^2 2x - x^2\,dx$

$= \frac{1}{2}\left[x^2 - \frac{1}{3}x^3\right]_0^2 = \frac{1}{2}\left[4 - \frac{8}{3}\right] = \frac{1}{\cancel{2}}\left(\frac{\cancel{4}^{2}}{3}\right) = \boxed{\frac{2}{3}}$

$\frac{12}{3}$

New 11. What is the slope of the polar curve $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$?

$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$ $r' = -2\sin\theta$

$= \frac{-2\sin^2(\frac{\pi}{2}) + \cancel{(3 + 2\cos\frac{\pi}{2})\cos\frac{\pi}{2}}}{-2\sin\frac{\pi}{2}\cancel{\cos\frac{\pi}{2}} - (3 + 2\cos\frac{\pi}{2})\sin\frac{\pi}{2}} = \frac{-2}{-3} = \boxed{\frac{2}{3}}$

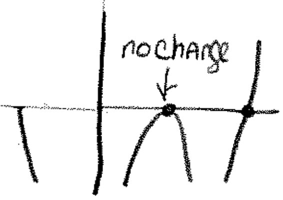
Typos #12

BC

✓ 12. **Calculator**

The velocity of a particle moving along the x-

$$v(\cancel{x}) = x^4 - 3x^3 - 9$$



~~How many~~ times does the particle change direction as X increases from −5 to 5?

$\boxed{\text{2 times}}$ ↖ velocity changes signs.

✓ 13. The circumference of a circle is increasing at a rate of $\frac{2\pi}{5}$ inches per minute. When the radius is 5 inches, how fast is the area of the circle increasing in square inches per minute?

$\frac{dC}{dt} = \frac{2\pi}{5}$ in/min

$r = 5$

$\frac{dA}{dt} = ?$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\cancel{\frac{1}{2\pi}} \cdot \frac{\cancel{2\pi}}{5} = \cancel{2\pi} \frac{dr}{dt}$

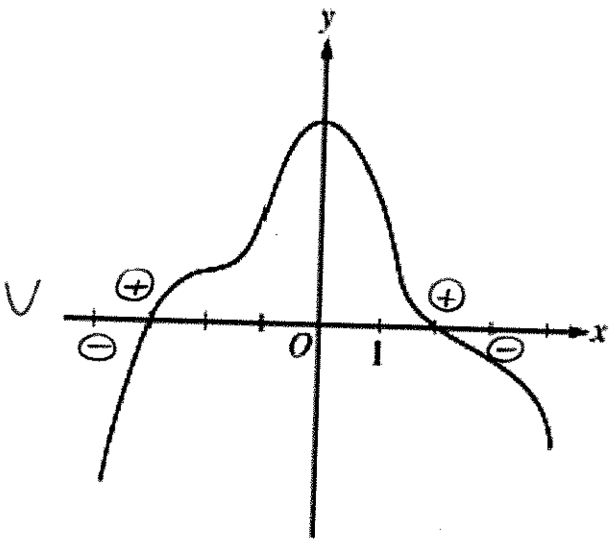
$\frac{1}{5} = \frac{dr}{dt}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi (\cancel{5})(\frac{1}{\cancel{5}}) = \boxed{2\pi}$ in²/min

✓ 14.



Graph of f'

The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$. ✓

II. The graph of f has a point of inflection at $x = -2$. X 2nd deriv would change signs

III. The graph of f is concave down for $0 < x < 4$. ✓ 2nd deriv < 0

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

✓ SLOPE

15. Find the instantaneous rate of change of $f(t) = (2t^3 - 3t + 4)\sqrt{t^2 + 3t + 4}$ at $t = 0$.

$f'(t) = (6t^2 - 3)\sqrt{t^2+3t+4} + (2t^3-3t+4)\cdot\frac{1}{2}(t^2+3t+4)^{-1/2}\cdot(2t+3)$

$f'(0) = (-3)\sqrt{4} + (\cancel{4})^{2}\cdot\cancel{\tfrac{1}{2}}(4)^{-1/2}(3)$ $\quad 1/4^{1/2}$

$-6 + 3$

$\boxed{-3}$

✓

16. Let f be the function defined by $f(x) = x^3 - 3x^2$ What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over $[0,3]$? $y' = \frac{f(b)-f(a)}{b-a}$ MVT

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

$3x^2 - 6x = \frac{0-0}{3-0}$ $\quad 3x^2-6x=0$

$3x(x-2)$ $\quad$ CAnnot include endpts.

$\cancel{x=0}$ $\quad x=2$

✓

17. Find the equation of the normal line to the curve $y = \sqrt{16 - x}$ at the point (0,4).

$y' = -\frac{1}{2}(16-x)^{-1/2}$

$y'(0) = -\frac{1}{2}(16)^{-1/2} = -\frac{1}{8}$

tAn line

$\boxed{y - 4 = 8x}$ or $y = 8x + 4$

New

18. Find the area of the region enclosed by the polar curve $r = cos(3\theta)$ for $0 \leq \theta \leq \pi$. ~~k~~

$A = \frac{1}{2}\int r^2 d\theta$

$A = \frac{1}{2}\int_0^{\pi}(\cos 3\theta)^2\, d\theta = \boxed{.785}$ or $\frac{\pi}{4}$

Arc Length

19. Write an integral that can be used to find the length of the curve $y = sin3x$ from $x = 0\ to\ x = 4$.

not polar

$S = \int_0^4 \sqrt{1 + (3\cos 3x)^2}\, dx$ $\quad y' = 3\cos 3x$

or

$\boxed{\int_0^4 \sqrt{1 + 9\cos^2(3x)}\, dx}$

✓

17 20. Find the value of each of the following:

a) $\int_1^{\infty} \frac{1}{x^4} dx.$ $\Rightarrow \lim_{a\to\infty} \int_1^a x^{-4}\,dx = -\frac{1}{3}x^{-3}\Big|_1^a = \lim_{a\to\infty} -\frac{1}{3}\left[\frac{1}{a^3} - 1\right]$

$-\frac{1}{3}[0 - 1] = \boxed{\frac{1}{3}}$

b) $\int_{-\infty}^{\infty} \frac{2x}{(x^2+1)^2} dx$ $\lim_{a\to-\infty} \int_a^0 \frac{1}{u^2}\,du + \lim_{b\to\infty} \int_0^b \frac{1}{u^2}\,du$

Let $u = x^2+1$

$du = 2x$

$-\frac{1}{x^2+1}\Big|_a^0 + \frac{-1}{x^2+1}\Big|_0^b = \left[-1 + \underbrace{\frac{1}{a^2+1}}_{0}\right] + \left[\underbrace{-\frac{1}{b^2+1}}_{0} + 1\right]$

$-1 + 1 = \boxed{0}$

c) $\int_0^3 \frac{dx}{x-1}$ $= \lim_{a\to 1} \int_0^a \frac{1}{x-1}\,dx + \lim_{a\to 1} \int_a^3 \frac{1}{x-1}\,dx$

VA $x=1$

$\ln|x-1|\Big|_0^a + \ln|x-1|\Big|_a^3$

$\left[\underbrace{\ln|a-1|}_{-\infty} - \ln 1\right] + \left[\ln 2 - \underbrace{\ln|a-1|}_{-\infty}\right]$

$\boxed{\text{DIVERGES}}$

Arc length

21. Which of the following gives the length of the path described by the parametric equations $x(t) = 2 + 3t$ and $y(t) = 1 + t^2$ from $t = 0$ to $t = 1$?

$x'(t) = 3$

$y(t) = 2t$

$S = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

(A) $\int_0^1 \sqrt{1 + \frac{4t^2}{9}}\, dt$

(B) $\int_0^1 \sqrt{1 + 4t^2}\, dt$

(C) $\int_0^1 \sqrt{3 + 3t + t^2}\, dt$

(D) $\int_0^1 \sqrt{9 + 4t^2}\, dt$

(E) $\int_0^1 \sqrt{(2 + 3t)^2 + \left(1 + t^2\right)^2}\, dt$

Let $u = x^2+4$, $du = 2x$

$\frac{1}{2}\int_0^k \frac{1}{u}\,du = \frac{1}{2}\ln(x^2+4)\Big|_0^k$

$\cancel{\frac{1}{2}}[\ln(k^2+4) - \ln 4] = \cancel{\frac{1}{2}}\ln 4$

✓22. If $\int_0^k \frac{x}{x^2+4}\,dx = \frac{1}{2}\ln 4$, where $k > 0$, then $k =$

$\ln k^2+4 = 2\ln 4$

$\ln k^2+4 = \ln 16$

$k^2+4 = 16$

$k^2 = 12$

$k = \sqrt{12}$

(A) 0 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{12}$ (E) $\frac{1}{2}\tan\left(\ln\sqrt{2}\right)$

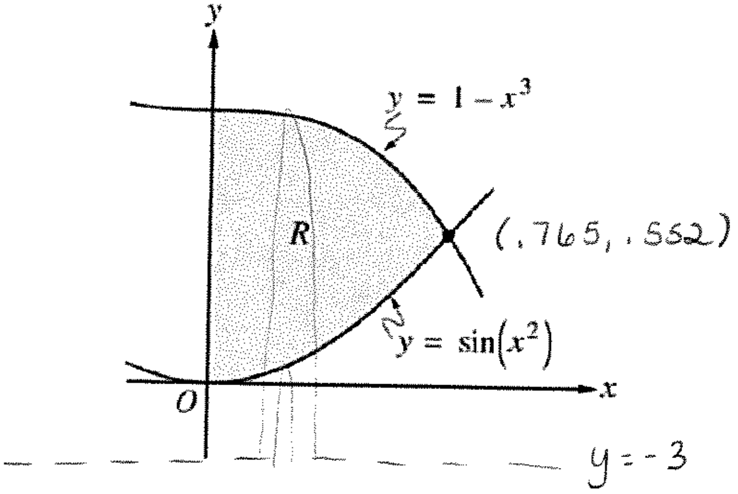
✓23. If $x = t^2 - 1$ and $y = \ln t$, what is $\frac{d^2y}{dx^2}$ in terms of t?

$\frac{dy}{dx} = \frac{\frac{1}{t}}{2t} = \frac{1}{t}\cdot\frac{1}{2t} = \frac{1}{2t^2} = \frac{1}{2}t^{-2}$

(A) $-\frac{1}{2t^4}$ (B) $\frac{1}{2t^4}$ (C) $-\frac{1}{t^3}$ (D) $-\frac{1}{2t^2}$ (E) $\frac{1}{2t^2}$

$\frac{d^2y}{dx^2} = \frac{-t^{-3}}{2t} = -\frac{1}{t^3}\cdot\frac{1}{2t} = \frac{-1}{2t^4}$

✓24. **CALCULATOR**



Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$, as shown in the figure above.

(a) Find the area of R. $A = \int_0^{.765} (1-x^3) - \sin(x^2)\,dx = \boxed{.534}$

b) Find the volume of the solid generated when R is revolved about the line $y = -3$.

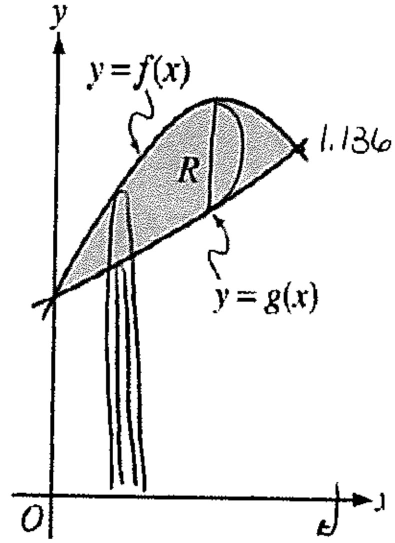
Washer Method

$V = \pi\int_0^{.765} (1-x^3-(-3))^2 - (\sin(x^2)-(-3))^2\,dx$

$= \boxed{3.769\pi}$ or $\boxed{11.841}$

✓ 25. **CALCULATOR**

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.



(a) Find the area of R. $\int_0^{1.136} [(1 + \sin 2x) - e^{x/2}]\, dx = \boxed{.429}$

washer (b) Find the volume of the solid generated when R is revolved about the
Method x-axis. $V = \pi \int_0^{1.136} (1+\sin 2x)^2 - (e^{x/2})^2\, dx = \boxed{4.267}$

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

$A = \frac{1}{2}\pi r^2$

$r = \frac{f(x) - g(x)}{2}$

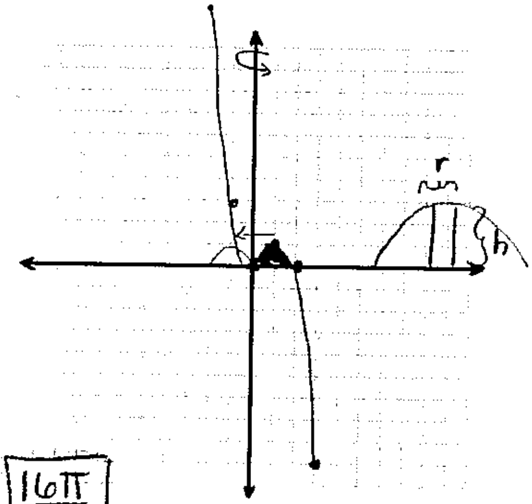
$$V = \frac{\pi}{2} \int_0^{1.136} \left(\frac{1 + \sin 2x - e^{x/2}}{2} \right)^2 dx$$

$$= \boxed{.078}$$

✓

26. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 2x^2 - x^3$ and $y = 0$ about the $y - axis$.

* w/ respect to y too difficult to get $x =$ $(x=0)$



Shell Method

$V = 2\pi \int r h\, dx$

$r = x$
$h = 2x^2 - x^3$

NO CALC

$V = 2\pi \int_0^2 x\,(2x^2 - x^3)\,dx$

$= 2\pi \int_0^2 2x^3 - x^4\, dx$

$2\pi \cdot \frac{1}{2}x^4 - \frac{1}{5}x^5 \Big|_0^2 = 2\pi\left[8 - \frac{32}{5}\right] = 2\pi \cdot \frac{8}{5} = \boxed{\frac{16\pi}{5}}$ $\frac{40}{5}$

✓

27. For $0 \le t \le 13$, an object travels along an elliptical path given by the parametric equations $x = 3cost$ and $y = 4sint$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

$\frac{dy}{dx} = \frac{4\cos t}{-3\sin t}$ $\quad \frac{dy}{dx}\Big|_{t=13} \frac{4\cos 13}{-3\sin 13} = \boxed{-\frac{4}{3}\cot 13}$ or $\frac{-4}{3\tan 13}$

no calculator

✓ 28. **CALCULATOR**

A particle moves in the $xy-plane$ so that its position at any time t is given by $x(t) = t^2$ and $y(t) = sin(4t)$. What is the speed of the particle when $t = 3$?

$$Sp = \sqrt{(2t)^2 + (4\cos 4t)^2}$$

$$= \sqrt{36 + 16\cos^2 12} = \boxed{6.8843}$$

✓ 29. For any time $t \geq 0$, if the position of a particle in the xy-plane is given by $x = t^2 + 1$ and $y = \ln(2t+3)$, then the acceleration vector is $\quad v = \left(2t, \frac{2}{2t+3}\right) \quad a = \left(2, \frac{-2 \cdot 2}{(2t+3)^2}\right)$

2nd deriv

(A) $\cancel{\left(2t, \frac{2}{(2t+3)}\right)}$ (B) $\cancel{\left(2t, \frac{-4}{(2t+3)^2}\right)}$ (C) $\left(2, \frac{4}{(2t+3)^2}\right)$

(D) $\left(2, \frac{2}{(2t+3)^2}\right)$ (E) $\left(2, \frac{-4}{(2t+3)^2}\right)$

✓ 30. The position of a particle moving in the $xy-plane$ is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

$x'(t)$ & $y'(t) = 0$

(A) -1 only (B) 0 only (C) 2 only (D) -1 and 2 only (E) -1, 0, and 2

$x'(t) = 3t^2 - 6t$ $\quad y'(t) = 6t^2 - 6t - 12$

$3t(t-2)$ $\quad 6(t^2 - t - 2)$

$t=0 \quad t=2$ $\quad (t-2)(t+1)$

$t=2 \quad t=-1$

CALC

31. For $0 \leq t \leq 3$, an object moving along a curve in the xy–plane has position $(x(t), y(t))$ with $\frac{dx}{dt} = \sin(t^3)$ and $\frac{dy}{dt} = 3\cos(t^2)$. At time $t = 2$, the object is at position (4,5). Find the position of the object at time $t = 3$.

$$x(3) = 4 + \int_2^3 x'(t)\,dt = 4 + .004 = \boxed{4.004}$$

$$y(3) = 5 + \int_2^3 y'(t)\,dt = 5 + .724 = \boxed{5.724}$$

2007 Form B
CALCULATOR

32. An object moving along a curve in the $xy-plane$ is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = arctan\left(\frac{t}{1+t}\right) \; and \; \frac{dy}{dt} = ln(t^2+1)$$

for $t \geq 0$. At time $t = 0$, the object is at position (-3,-4).

(a) Find the speed of the object at time $t = 4$.
(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.
(c) Find $x(4)$.
(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

a) Speed $= \sqrt{(\arctan(4/5))^2 + (\ln 17)^2} = \boxed{2.912}$

b) Total Dist. $= \int_0^4 \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt = \boxed{6.423}$

c) $x(4) = x(0) + \int_0^4 dx/dt \, dt$

$-3 + 2.108 = \boxed{-.892}$

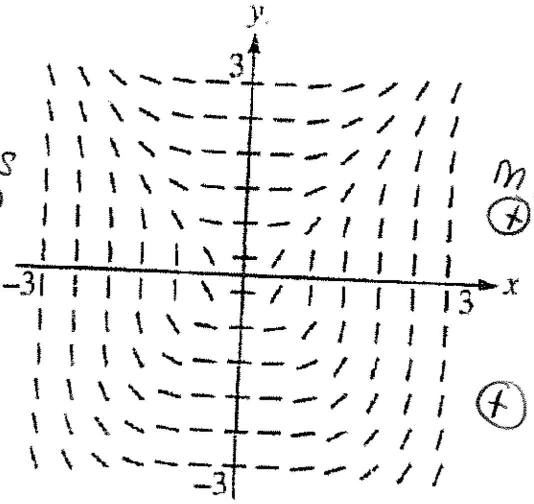
$t > 0$

d) $\frac{dy}{dx} = \frac{\ln(t^2+1)}{\arctan(\frac{t}{1+t})} = 2$

$\boxed{t \approx 1.358}$

accel $= \boxed{< .135, .955 >}$

✓ 33.



m is ⊖ so when x⊖ → m⊖

m is ⊕

when x+y⊖ m=⊖

⊕

m = 0 when x = 0

Shown above is a slope field for which of the following differential equations?

(A) $\cancel{\frac{dy}{dx} = \frac{x}{y}}$ (B) $\cancel{\frac{dy}{dx} = \frac{x^2}{y^2}}$ (C) $\frac{dy}{dx} = \frac{x^3}{y}$ (D) $\cancel{\frac{dy}{dx} = \frac{x^2}{y}}$ (E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

Q3 this would be ⊕

Q2 would be ⊕

Q3 -/- would be ⊕

Q2 would be ⊕

$\frac{dP}{dt} = kP(L-P)$ $P = \frac{L}{1+Ce^{-Lkt}}$ (Lice - Licked)

LOGISTIC Growth

34. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t\to\infty} M(t)$?

SAME as P

max capacity

$\frac{.6}{200} M(200 - M)$ *Factor out 1/200

(K) (L)

(A) 50 (B) 200 (C) 500 (D) 1000 (E) 2000

35. **CALCULATOR**

Ten grizzly bears were introduced to a national park 10 years ago. There are 23 bears in the park at the present time. The park can support a maximum of 100 bears. Assuming a logistic growth model, when will the bear population reach 75?

L

$t=0$ $P=10$

$t=10$ $P=23$

$L=100$

$P=75$ $t=?$

$10 = \frac{100}{1+Ce^{-100k(0)}}$

$10+10C = 100$

$10C = 90$

$\boxed{C=9}$

$23 = \frac{100}{1+9e^{-100k(10)}}$

$100 = 23 + 207e^{-1000k}$

$\frac{77}{207} = e^{-1000k}$

$\boxed{k = \frac{\ln 77/207}{-1000}}$

$75 = \frac{100}{1+9e^{\frac{\ln 77/207}{10}t}}$

$100 = 75 + 675e^{\frac{\ln 77/207}{10}t}$

$\frac{1}{27} = e^{\frac{\ln 77/207}{10}t}$

$t = \frac{\ln 1/27}{\left(\frac{\ln 77/207}{10}\right)}$

$= \boxed{33.328 \text{ years}}$

36. **CALCULATOR**

A hard-boiled egg at 98°C is put in a pan under running 18°C water to cool. After 5 minutes, the egg's temperature is found to be 38°C. How much longer will it take the egg to reach 20°C?

$T - T_s = (T_0 - T_s)e^{-kt}$

* Its a T_{emp} S.O.S

$T_0 = 98$

$T_s = 18$

$t=5$ $T=38$

$t=?$ $T=20$

First

$38-18 = (98-18)e^{-5k}$

$\frac{1}{4} = e^{-5k}$

$\boxed{\frac{\ln 1/4}{-5} = k}$

Now

① $20-18 = (98-18)e^{\frac{\ln 1/4}{5}t}$

$\frac{1}{40} = e^{\frac{\ln 1/4}{5}t}$

$\ln 1/40 = \frac{\ln 1/4}{5}t$

$t = 13.305 - 5 = \boxed{8.305}$

or

② $20-18 = (38-18)e^{\frac{\ln 1/4}{5}t}$

$\frac{2}{20} = e^{\frac{\ln 1/4}{5}t}$

$\ln 1/10 = \frac{\ln 1/4}{5}t$

$\boxed{t = 8.305}$

✓ 37. **CALCULATOR**

Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton how many tons did the insects destroy during the time interval $7 \le t \le 14$?

$$\int_7^{14} \frac{100e^{-0.1t}}{2-e^{-3t}}\,dt = 124.994 = \boxed{125}$$

✓ 38. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^{x} + 5\right)$

(D) $y = -\ln\left(e^{x} + 3\right)$

(E) $y = \ln\left(e^{x} + 3\right)$

$\frac{dy}{dx} = e^{y} \cdot e^{x}$

$\int \frac{1}{e^{y}}\,dy = \int e^{x}\,dx$

e^{-y}

$\frac{-1}{e^{y}} = e^{x} + C$

$\frac{-1}{e^{-\ln 4}} = 1 + C$

$-e^{\ln 4}$

$-4 = 1 + C$

$-5 = C$

$\frac{-1}{e^{y}} = e^{x} - 5$

$e^{y}(e^{x}-5) = -1$

$\ln e^{y} = \ln \frac{-1}{e^{x}-5}$

$y = \ln \frac{-1}{e^{x}-5} = \ln -(e^{x}-5)^{-1}$

$-\ln(-e^{x}+5)$

✓ 39. Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the intial condition $f(1) = 4$.

$\int 2y\,dy = \int (3x^2+1)\,dx$

$y^2 = x^3 + x + C$

$16 = 1 + 1 + C$

$14 = C$

$y^2 = x^3 + x + 14$

$$\boxed{y = \sqrt{x^3 + x + 14}}$$

only ⊕ b/c ↑ pt.

37 ✓ 40.

x	2	3	5	8	13
$f(x)$	6	−2	−1	3	9

The function f is continuous on the closed interval [2, 13] and has values as shown in the table above. Using the intervals [2, 3], [3, 5], [5, 8], and [8, 13], what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

$1f(2) + 2f(3) + 3f(5) + 5f(8)$

$6 + (-4) + (-3) + 15 = \boxed{14}$

✓

41. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with the initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

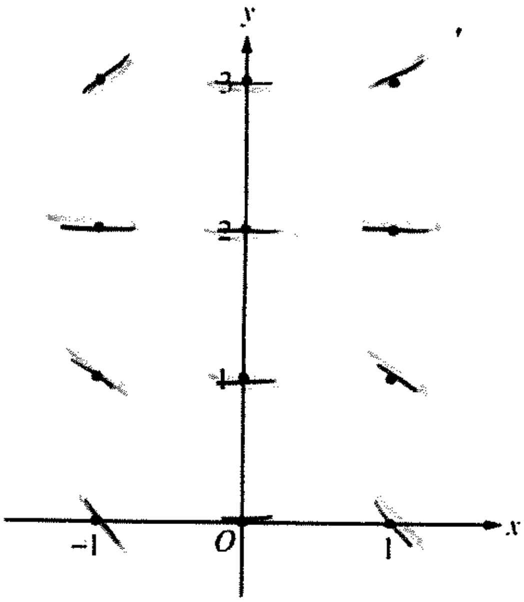
x	$\overset{\text{old}}{y} + \frac{dy}{dx}(\Delta x)$	y
1	—	0
3/2	$0 + 2(\frac{1}{2})$	1
2	$1 + 4(\frac{1}{2})$	3

$\boxed{f(2) = 3}$

2004 AB 42 FormB #5

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



Slopes are negative for all $y < 2$ where $x \neq 0$

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

$\int \frac{1}{y-2}\,dy = \int x^4\,dx$

$\ln|y-2| = \frac{1}{5}x^5 + C$

$\ln|-2| = C$

$\ln|y-2| = \frac{1}{5}x^5 + \ln|-2|$

$|y-2| = e^{1/5x^5 + \ln|-2|}$

$\boxed{y = \pm 2e^{1/5x^5} + 2}$

2001 BC #2

41

✓ 43. **CALCULATOR**

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period. avg rate of change

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

a) $W'(12) = \dfrac{W(15)-W(9)}{15-9} = \dfrac{21-24}{6} = \boxed{-\tfrac{1}{2}\ °C/day}$ or can use $\dfrac{W(15)-W(12)}{15-12}$ $\dfrac{W(12)-W(9)}{12-9}$

b) $3\left(\dfrac{20+31}{2} + \dfrac{31+28}{2} + \dfrac{28+24}{2} + \dfrac{24+22}{2} + \dfrac{22+21}{2}\right) = 3(125.5) = 376.5$

Avg. temp. $= \frac{1}{15}(376.5) = \boxed{25.1\ °C}$

Product Rule

c) $P'(t) = 10e^{-t/3} + 10t\,e^{-t/3} \cdot -\frac{1}{3}$

$P'(12) = \dfrac{10}{e^4} + \dfrac{\cancel{120}\ 40}{e^4} \cdot -\dfrac{1}{\cancel{3}} = \boxed{\dfrac{-30}{e^4}}$ or $-.549$

The temperature of the water is decreasing at a rate of .549 °C/day when $t = 12$.

d) $\frac{1}{15}\int_0^{15} P(t)\,dt = \boxed{25.757°C}$

avg. daily temp.

38

44. **CALCULATOR**

The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

(a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

Abs. max

(b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

t y

(c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

a) $S = \int_0^4 E(t)\,dt = \boxed{3981 \text{ gallons}}$

b) $A(x) = \int_0^t \underset{\text{in}}{E(x)\,dx} - \underset{\text{out}}{645x}$

$A'(t) = E(t) - 645 = 0$ when $t = 2.309$ and 3.559

FTC

x	y
0	0
2.309	A(2.309) = 1637
3.559	A(3.559) = 1229
4	3981 - 645(4) = 1401

$\boxed{\text{Amount of sewage in tank is a maximum at } t = 2.309 \text{ when there is 1637 gallons of sewage.}}$

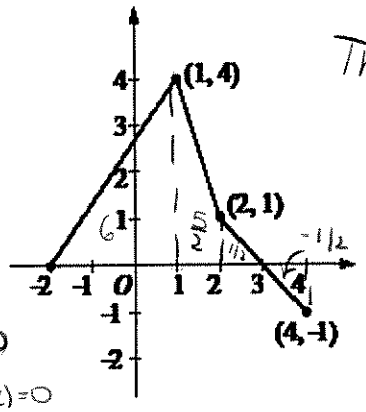
c) Cost at t is $(0.15 - 0.02t)E(t)$

Total cost $= \int_0^4 (0.15 - 0.02t)\,E(t)\,dt = 474.320$

$\boxed{\text{Total cost, to the nearest dollar is \$474.}}$

39

45. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.



(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

Critical values at $g''(x)=0$ or $g''(x)$ DNE

a) $g(4) = \int_1^4 f(t)\,dt = \frac{5}{2} + \frac{1}{2} - \frac{1}{2} = \boxed{\frac{5}{2}}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt = \boxed{-6}$

b) $g'(x) = f(x)$

$g'(1) = f(1) = \boxed{4}$

c) $g'(x) = f(x) = 0$ when $x = -2$ and $x = 3$

x	y
-2	-6
3	$\int_1^3 f(t)\,dt = 3$
4	5/2

Abs minimum value is -6 which occurs at $x = -2$.

Critical values are where deriv = 0 or DNE

d) on $[-2, 1]$ $g''(x) = f'(x) > 0$

on $[1, 2]$ $g''(x) = f'(x) < 0$

on $[2, 4]$ $g''(x) = f'(x) < 0$

So the only sign change occurs at $x = 1$ therefore $(1, g(1))$ is the only pt of infuction. $\boxed{\text{ONE}}$

46. $\int_1^e \frac{x^2+1}{x}\,dx = \int_1^e x + \frac{1}{x}\,dx = \frac{1}{2}x^2 + \ln x \Big|_1^e$

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+1}{2}$ (C) $\frac{e^2+2}{2}$ (D) $\frac{e^2-1}{e^2}$ (E) $\frac{2e^2-8e+6}{3e}$

$= \frac{e^2}{2} + \ln e - (\frac{1}{2} + \cancel{\ln 1}) = \frac{e^2}{2} + \frac{1}{2}$

HO

✓ 47. a) $\lim_{x\to 1} \frac{\int_1^x e^{t^2}\,dx}{x^2-1}$ $\frac{0}{0}$ $\lim_{x\to 1} \frac{e^{x^2}}{2x} = \boxed{\frac{e}{2}}$

L'hopital

b) $\lim_{x\to 0}(1+6x)^{\csc x} = 1^{\infty}$ $\lim_{x\to 0} \frac{\ln(1+6x)}{\sin x} = \frac{0}{0}$

$\lim_{x\to 0} \frac{\frac{6}{1+6x}}{\cos x} = \frac{6}{1} = 6 \Rightarrow \boxed{e^6}$

✓ 48.

x	0	1	2	3
$f''(x)$	5	0	−7	4

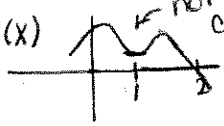
horizontal tangent

The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

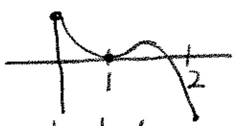
(A) f is increasing on the interval $(0, 2)$. X $\overset{(0,1)}{\text{incr}}$ then $\overset{(1,2)}{\text{decr.}}$ $\cap$

(B) f is decreasing on the interval $(0, 2)$. X

$f'(x)$ not a critical value of f $f'(x) \neq 0$



(C) f has a local maximum at $x = 1$.



(D) The graph of f has a point of inflection at $x = 1$. Could be but not def. more info. needed

(E) The graph of f changes concavity in the interval $(0, 2)$. b/c 2nd deriv goes from ⊕ to ⊖ somewhere

✓ 49. On the graph of $y = f(x)$, the slope at any point (x, y) is twice the value of x. If $f(2) = 3$, what is the value of $f(3)$?

initial cond (2, 3)

slope $f'(x) = 2x$

$f(x) = \int 2x\,dx = x^2 + C$ $f(x) = x^2 - 1$

$3 = 4 + C$ $\boxed{f(3) = 8}$

$-1 = C$