Parametric Equations Review

CALCULATOR ALLOWED

Directions: Read each question carefully and show ALL work.

1. What is the slope of the line segment, in the xy −plane, given by the parametric equations $x = 3t + 4$ and $y = 7t - 1$, for $-2 \leq t \leq 5$.

2. The velocity of a particle moving in the xy −plane at any time , $0 \leq t \leq 2\pi$, is given by the parametric equations $\frac{dx}{dt} = cost$ and $\frac{dy}{dt} = -2\sin(2t)$. For what value(s) of t is the particle at rest?

3. The velocity of a particle moving on a curve is given by, $v = \langle cos(3t), e^{2t} \rangle$. When $t = 0$, the particle is at point (0,1).

   (a) Find the position vector at time t.
   (b) Find the acceleration vector at time t.

4. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \cos(t^3) \text{ and } \frac{dy}{dt} = 3\sin(t^2)$$

for $0 \le t \le 3$. At time $t = 2$, the object is at position $(4,5)$.

(a) Write an equation for the line tangent to the curve at $(4,5)$.

(b) Find the speed of the object at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

(d) Find the position of the object at time $t = 3$.

5. Find the point at which the tangent to the curve defined parametrically by $x = 4cost$ and $y = 9sint$, is (a) horizontal; (b) vertical.

6. Find the length of the curve given by $x = \frac{1}{3}t^3$ *and* $y = \frac{1}{2}t^2$ *from* $0 \leq t \leq 4$.

7. Write an integral that can be used to find the length of the path described by the parametric equations $x = 2cos^3 t$ and $y = 2sin^3 t$ for $0 \leq t \leq 2\pi$.

8. The path of a particle is given for the time $t \geq 0$ is given by $x = t^2 - 2$ and $y = \frac{2}{5}t^3$.

   (a) Find $\frac{dy}{dx}$ when $t = 2$.

   (b) Find $\frac{d^2y}{dx^2}$ when $x = 7$.

9.

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

10. Write the equation of the tangent line to the graph of $r = 4\theta$ at the point $\left(2, \frac{3\pi}{2}\right)$.

11. Find any point(s) at which the tangent to the curve defined by $r = 1 + sin\theta$ from $0 \leq \theta \leq 2\pi$ is

(a) horizontal

(b) vertical.

12) A curve in the plane is defined parametrically by the equations $x = \frac{1}{2}t^3 - 6t$ and $y = 2t^4 + 3t^2$. An equation of the line normal to the curve at $t = 1$ is

Answers

1. $7/3$ 2. $t = \frac{\pi}{2}, \frac{3\pi}{2}$ 3.a) $\langle \frac{1}{3}sin(3t), \frac{1}{2}e^{2t} + \frac{1}{2} \rangle$ b) $\langle -3sin(3t), 2e^{2t} \rangle$

4.a) $y - 5 = 15.604(x - 4)$ b) 2.275 c) 1.458 d) (3.954, 4.906)

5.a) (0,9) (0,-9) b) (4,0) (-4,0) 6. $\frac{17\sqrt{17} - 1}{3}$

7. $\int_0^{2\pi} \sqrt{36cos^2tsin^2t}\ dt$ *or* $\int_0^{2\pi} 6|costsint|dt$ 8.a) 6/5 b) 1/10

9.a) 7.133 b) $y - 8 = -2.983(x - 1)$ c) 7.383 d) $\langle 2.303, 24.814 \rangle$

10. $y + 2 = -\frac{2}{3\pi}x$

11.a) $\left(2, \frac{\pi}{2}\right), \left(\frac{1}{2}, \frac{7\pi}{6}\right), \left(\frac{1}{2}, \frac{11\pi}{6}\right)$ b) $\left(\frac{3}{2}, \frac{\pi}{6}\right)\left(\frac{3}{2}, \frac{5\pi}{6}\right)$

12. $y - 5 = \frac{9}{28}(x + \frac{11}{2})$